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# MIMS – Diversified Passive Selection Fund

## Portfolio Management Team

### Report – May 2023

**Fund Description**

The Passive Fund is composed by a number of Exchange Traded Funds selected by Minerva Investment Management Society, reflecting the output of the research of the Passive Portfolio Team. These ETFs aim to replicate as closely as possible the performance of a basket of securities with specific common properties, thus being effective instruments for investors who wish to express a certain view on industry sectors or economic trends while capturing as little idiosyncratic risk as possible. Each ETF was carefully chosen in line with the macroeconomic outlook. Our allocation is based on a diversification process achieved among geographies, asset classes and sectors.

## Allocation Breakdown

**Asset Allocation**

As in the previous semester, we overweigh fixed income with a strong focus on short duration and high-quality bond ETFs. We think that this asset class has finally resurfaced from the depths of the historical LIRE and is now more appealing to investors. Our lower weight in equity is especially due to the current uncertainty and recession expectations, which could entail pain for the asset class in the following months. To contain eventual drawdowns, our strategy revolves around defensive and fixed income ETFs. With regards to commodities, we implement a conservative allocation but distance ourselves from fossil fuels. Our allocation is thus split in fixed income (55%), equity (30%) and commodities (15%).

**Geographical Allocation**

We took a defensive view on the allocation in US and Europe due to the adverse macroeconomic environment waiting ahead in our opinion. Hence, we prefer to position ourselves globally on multiple asset classes to avoid any local shock or eventual distress.

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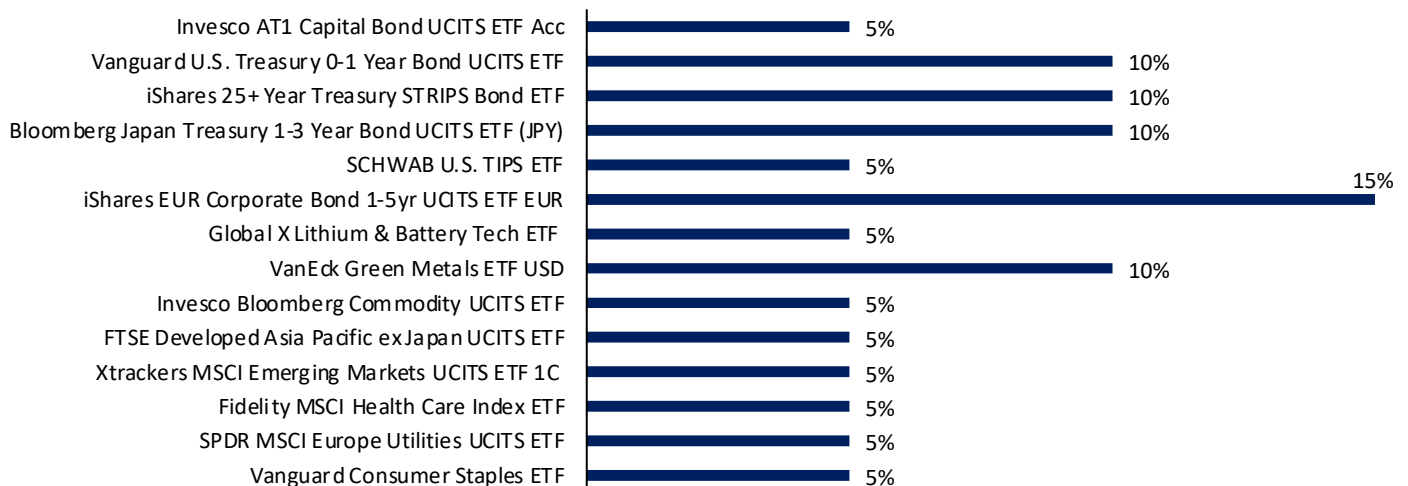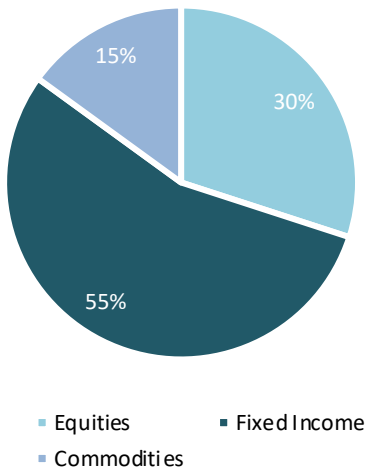Andrei Tudoroiu: andrei.tudoroiu@studbocconi.it
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As for Emerging Markets, we underweight China due to future expected decline in performance, prolonged uncertainty of data and geopolitical tensions on the Taiwan front. Exposure to Japan is overweighted mainly because of a positive outlook regarding the Yen, which is still standing near historical lows. We maintain unchanged our view on other Emerging Markets.
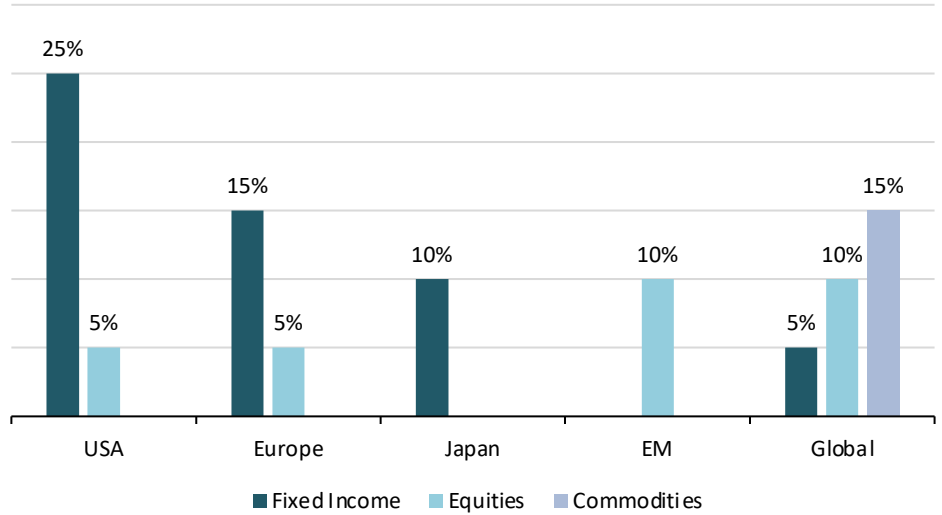
**ETFs Breakdown**

| ETF | Weight |
|---|---|
| Invesco AT1 Capital Bond UCITS ETF Acc | 5% |
| Vanguard U.S. Treasury 0-1 Year Bond UCITS ETF | 10% |
| iShares 25+ Year Treasury STRIPS Bond ETF | 10% |
| Bloomberg Japan Treasury 1-3 Year Bond UCITS ETF (JPY) | 10% |
| SCHWAB U.S. TIPS ETF | 5% |
| iShares EUR Corporate Bond 1-5yr UCITS ETF EUR | 15% |
| Global X Lithium & Battery Tech ETF | 5% |
| VanEck Green Metals ETF USD | 10% |
| Invesco Bloomberg Commodity UCITS ETF | 5% |
| FTSE Developed Asia Pacific ex Japan UCITS ETF | 5% |
| Xtrackers MSCI Emerging Markets UCITS ETF 1C | 5% |
| Fidelity MSCI Health Care Index ETF | 5% |
| SPDR MSCI Europe Utilities UCITS ETF | 5% |
| Vanguard Consumer Staples ETF | 5% |

## Asset Allocation



- Equities
- Fixed Income
- Commodities

## Geographical Breakdown



- Fixed Income
- Equities
- Commodities

## Performance

### Portfolio Value



| Period Return 01/12/2022 - 28/04/2023 | -4.645% |
|---|---|

| | |
|---|---|
| Daily Mean Return | -0.04% |
| Daily Volatility | 0.44% |
| Annualized Mean Return | -11.30% |
| Annualized Volatility | 7.14% |

In order to assess the performance of our investments, we track the daily value of the portfolio over a period of time stretching from December 2022 to April 2023. At the beginning of the observed period (December 1, 2022), we assume an initial investment of €10,097,000 and calculate the number of shares of each ETF that will be bought and held in portfolio, according to the weights chosen during the asset allocation process. Keeping track of the funds' prices, we can determine the value of the portfolio until the end of the period (April 28, 2023). We record a final value of 9,627,957.57 €, with an overall return of approximately -4.65% in approximately 6 months.

## Portfolio Overview

Our portfolio can be ideally divided into three parts:
- Equity Indexes ETFs
- Fixed Income ETFs
- Commodities ETFs

## Equity Indexes ETFs

Like in last semester, we underweight our equity component of the fund by another 15%. It now represents 30% of the total allocation, split across six ETFs covering different geographic areas: US, Europe, Emerging Markets and Japan. Our decision to decrease the allocation revolves around the problematic macroeconomic environment that has developed for the majority of 2023. The S&P 500 index's behavior has been challenging to understand since the beginning of the year. At first glance, the S&P 500's rise of more than 8% appears to indicate a reassuring picture of the US economy and investor sentiment. However, the significant disparity in performance between the top 5 or 10 largest capitalizations and the rest of the index paints a completely different picture, suggesting that the economy is not completely well-off. The Russell 2000 index of small and mid-cap stocks, which best represents domestic economic activity, has remained almost unchanged since the start of the year.

From a fundamental perspective, we believe that we are currently in a sensitive phase of the economic cycle. Following several quarters of rate hikes, US and European economies have entered a precarious phase. The S&P 500's earnings growth has been negative for the fourth consecutive quarter, and valuation levels are far from being cheap. The risk premium offered by risky assets appears inadequate, given the interest rates offered by cash and the yields to maturity on investment-grade bonds.

While the Fed is close to the end of rate hiking cycle, a pivot would cause macro deterioration, therefore we expect an Equity Risk Premium re-rating. Hence, we decided to implement a defensive equity strategy targeting Consumer Staples, Utilities and the Pharmaceutical sector. We believe that these industries will provide the appropriate protection we need through a more stable performance throughout a possible recession. Finally, as a thematic exposure we choose Lithium Batteries, having a strong long-term tailwind.

## Fixed Income ETFs

We decided to increase our fixed income exposure from 30% to 50% of the total allocation. Our fixed income component is spread across six ETFs, covering different geographic areas, and denominated in three currencies, to guarantee optimal risk diversification. The ETFs included in our portfolio contain investment-grade government and corporate bonds, characterized by short- and medium-term maturity, ensuring less volatility to changes in interest rates.

This asset class is the main driver of the portfolio's performance, coherently with our conservative view. High-credit investment grade bonds and banks' AT1 offer extremely good yields that will generate a steady, recurrent return over the coming months.

We included different types of ETFs that will let us gain exposure to several macroeconomic factors, such as Japanese Government Bonds, confirming our bullish view on the country's currency. Finally, we included long-term zero-coupon bonds as a hedge against a possible severe recession (which will probably trigger a quick reduction in interest rates).

## Commodities (Hedging) ETFs

The commodities market is shaped by a lot of factors, from climate uncertainties and technological breakthroughs to policy reforms and shifts in consumer behavior. Despite formidable performance in the last year, we are taking a more conservative approach on commodities once again, reducing its weight to 15%. This semester, commodities will serve as a major hedge and element of diversification within the portfolio.

For energy commodities, we expect that a lower pricing trend is anticipated to continue, driven by a predicted decline in global demand as well as a greater demand for renewable energies.

On the other hand, the persistently high prices of fertilizers continue to have a big impact on the world's food supply, inflation, and cost of living in general. The agricultural market provides a brief moment of calm, projecting a modest decrease in prices over the forthcoming year. Nevertheless, the near-record agricultural prices continues to underscore the urgent matter of worldwide food security and food inflation.

## iShares EUR Corporate Bond 1-5yr UCITS ETF EUR

Index: Bloomberg Euro Corporate 1-5 Year Bond Index

Expense Ratio: 0.20%. Tracking Error Volatility: 1.41%

### Overview
The iShares EUR Corporate Bond 1-5yr UCITS ETF EUR tracks the Bloomberg Euro Corporate Bond 1-5 index. The index tracks Euro denominated investment grade corporate bonds from industrial, utility and financial issuers publicly issued in the Eurobond and eurozone domestic markets.

### Analysis
In our opinion, the ETF brings in yield by investing in bonds issued by companies in different sectors.

These short and medium-term bonds became very attractive in recent times, thanks to rising interest rates and therefore better yields. The companies that are emitting these bonds are highly rated and considered safe. We preferred to gain more exposure to the Euro as we find yields on bonds in the Eurozone more attractive.



### Conclusion
The ETF has returned +0.90% YTD. This is a solid investment given the current macroeconomic scenario. Safe investments such are bonds are now offering very interesting returns. We believe that this is still time to avoid exposure to equities and prefer conservative investments. Combining these two aspects, we believe this is a sound investment.

## Schwab US TIPS ETF

Index: Bloomberg US Treasury Inflation-Linked Bond Index

Expense Ratio: 0.04%. Tracking Error Volatility: 0.41%

### Overview
The fund's goal is to track the total return of an index composed of inflation-protected US Treasury securities. The expense ratio is low, and the type of instrument is a great asset to balance a portfolio in inflationary times.

### Analysis
The ETF tracks the performance of US TIPS (Treasury Inflation-Linked Bonds). This type of fixed-income instrument has the characteristic of hedging against inflation. While classic bonds pay interest on a fixed face value and are therefore sensitive to changes in inflation rates, TIPS have adjustable face values that track changes in inflation, therefore granting that the value of the interest payments are not eroded by inflation.
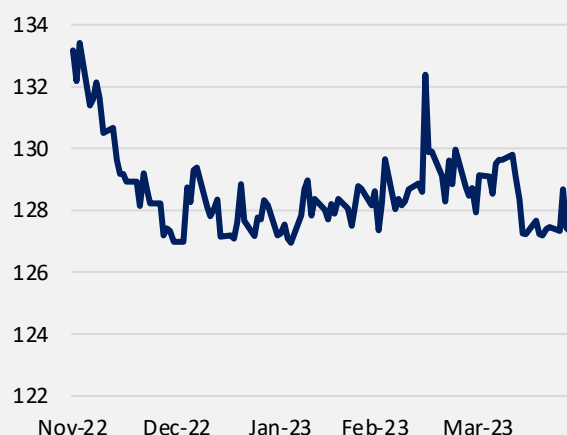


### Conclusion
The ETF has returned +3.45% YTD. The market for TIPS has attracted many conservative investors over the past year, providing stability and eliminating uncertainties. While we believe that the inflationary environment will start easing, the next sixth months will likely still be characterized by uncertainty. Therefore, we want to position ourselves to hedge against this risk.

## Vanguard Consumer Staples ETF

Index: MSCI US Investable Market Consumer Staples 25/50 Index

Expense Ratio: 0.10%. Tracking Error Volatility: 0.02%

**Overview**

This ETF tracks the performance of the MSCI World Consumer Staples index. It invests in consumer staples stocks from the entire globe.

**Analysis**

This ETF is exposed to the Consumer Staples sector. Indeed, its biggest holding include wholesalers like Costco and Walmart, consumer discretionaries like Nestle and Coca-Cola. We take an overweight vision on the sector in order to be positioned defensively. The rationale is that these types of companies tend to perform relatively well in adverse macroeconomic conditions due to their offering of essential consumer products, whose demand is unlikely to be heavily impacted by an economic slowdown. We strongly believe in ESG principles in the long term, but we still decided to opt for including tobacco companies due to the characteristics of its goods.



**Conclusion**

The ETF has returned 5.09% YTD. Rate hikes and high food prices damaged consumers and led them to limit their spending habits to non-discretionary goods. We believe that there is a significant chance of US falling into a recession in the next months. Therefore, we consider this pick a reliable choice.

## Invesco AT1 Capital Bond UCITS ETF Acc

Index: iBoxx® USD Contingent Convertible Liquid Developed Market AT1 index

Expense Ratio: 0.39% Tracking Error Volatility: -

**Overview**

This ETF invests in AT1 Convertible Bonds of European banks.

**Analysis**

During Credit Suisse's recent turmoil, these type of securities issued by CS have been written off due to a distortion in the hierarchy of capital redemption requests implemented by FINMA. Investors did not agree with FINMA's decision and are now demanding a refund on these securities. At the moment, several lawsuits are being filed and the scenario of a class action is becoming more and more plausible. This ETF expose us not only to this particular scenario, but also to a type of security that took a significant and excessive hit by the market given the stability and the solidity of most institutions, especially European ones.



**Conclusion**

For sure, the AT1 segment will still remain a source of volatility. However, given the recent tumble on these bonds and its particularly high yields, we find this investment really attractive from a return-risk perspective.

## SPDR MSCI Europe Utilities UCITS ETF

Index: MSCI Europe Utilities 35/20 Capped Index

Expense Ratio: 0.18%. Tracking Error Volatility: 0.19%

### Overview
The ETF seeks to track the performance of the MSCI Europe Utilities 35/20 Capped Index, designed to measure the performance of the large and mid-cap companies which operate in the utilities sector, across 15 countries in Europe.

### Analysis
In our attempt to select recession-resistant sectors for our equity allocation, we decide to confirm our exposure in this ETF. As previously explained last semester, it is very well-diversified across the range of companies of this sector. This ETF primarily includes companies that produce most of the energy they distribute. Their earnings are strongly correlated with inflation, making them the perfect sector in case of a sticky-inflation. Furthermore, many of the companies included in the ETF are leaders in the production of renewable energy, which will inevitably lead the industry sooner or later. This ETF ensures exposure to more solid industries which are likely to hold their ground during an economic recession.



### Conclusion
The ETF has returned a whopping 12% YTD. While this is already a solid performance, utilities valuations are not at their historical highs, and they will be beneficial of several public measures to sustain renewable energy production.

## Fidelity MSCI Health Care Index ETF

Index: MSCI US Health Care Index

Expense Ratio: 0.21%. Tracking Error Volatility: 0.17%

### Overview
The ETF replicates the MSCI US Health Care Index. This index reflects the performance of a wide array of leading healthcare firms, offering extensive exposure to multiple segments such as pharmaceuticals, biotechnology, and medical equipment.

### Analysis
The fund's emphasis on large-cap companies guarantees that it is well-diversified and less vulnerable to market shocks, by offering diverse exposure to numerous healthcare subsectors, mitigating the effects of fluctuations in individual stock prices. This feature makes it an appealing choice for investors seeking exposure defensive sectors. The ETF's largest allocations are in pharmaceuticals, biotechnology, and medical devices, providing a varied mix of healthcare companies. Furthermore, the healthcare industry has grown significantly in recent years, due to demographic trends, unexpected pandemics and rising demand for healthcare services.



### Conclusion
Advancements in medical research, increasing need for healthcare services due to ageing people, and the prospect of further innovation imply that the healthcare sector could present growth potential in the mid-to-long term.

## FTSE Developed Asia Pacific ex Japan UCITS ETF

Index: FTSE Developed Asia Pacific ex Japan Index

Expense Ratio: 0.2%. Tracking Error Volatility: 0.13%

### Overview

The FTSE Developed Asia Pacific ex Japan UCITS ETF offers investors access to a diversified portfolio of companies in the APAC region ex Japan. This ETF comprises 880 equities, providing broad exposure to the area's economies, sectors, and markets.

### Analysis

Key industries covered include finance, healthcare, technology, and consumer discretionary, with Taiwan, South Korea, and Hong Kong representing more than 60% of the geographic allocation. This exposes investors to some of the region's most vibrant and innovative businesses. In the current environment, marked by rapid technological advancements and increasing consumer spending, the ETF provides a potentially attractive investment opportunity for investors looking to benefit from the growth prospects of the region.



### Conclusion

The ETF has a 3.30% YTD return, reflecting a modest performance of companies within the developed APAC region. This moderate return can be attributed to various factors, including regional geopolitical tensions and economic uncertainties. However, with the central banks in the region maintaining accommodative monetary policies and ongoing government initiatives to support growth, there may be opportunities for a positive trend in the coming months.

## Xtrackers MSCI Emerging Markets UCITS ETF 1C

Index: MSCI Total Return Net Emerging Markets Index

Expense Ratio: 0.18%. Tracking Error Volatility: 1.73%

### Overview

The ETF mimics the performance of the MSCI Total Return Net Emerging Markets Index, designed to measure equity market performance of large and mid-cap companies in emerging markets. The fund has a global exposure, including prominent portions allocated in India, Taiwan, Brazil, South Korea and Saudi Arabia.

### Analysis

We believe that the portfolio could still benefit from an exposure to emerging markets, especially in the long-run. More specifically, the ETF is well-diversified from a geographical standpoint. Indeed, it has a limited exposure to China compared to similar funds on emerging markets, therefore reducing the negative impact of data uncertainty for the country. It has become more and more difficult to see what's really happening in the Chinese economy, due to the government effort to create a tightly sealed vault for its economy, cutting off international access to various databases.



### Conclusion

Emerging markets are known to be riskier than developed countries, therefore promising higher expected returns. However, the ETF's uncorrelation with the other securities of the portfolio reduces its overall systematic risk, making it an attractive choice.

## Bloomberg Japan Treasury 1-3 Year Bond UCITS ETF

Index: Bloomberg Global Japan Treasury 1-3 Year

Expense Ratio: 0.15%. Tracking Error Volatility: 0.01%
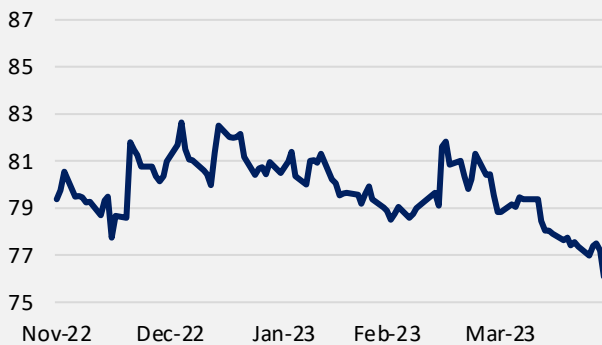
### Overview
The Bloomberg Global Japan Treasury 1-3 Year index tracks Japanese government bonds denominated in JPY.

### Analysis
With the change of the BoJ's last-longing governor, we expect the low interest rate cycle to end in the upcoming future. This ETF is the best to benefit from such event. The securities contained in this ETF are backed by the Japan Government.

The bonds' short duration will shield us from an abrupt decline in their prices in the case of rates hike, allowing us to benefit from the possible JPY appreciation at a low cost and with relatively low risk.

Furthermore, JPY is perceived as a safe-haven currency because of Japan's large current account surplus and stable economy. It has also a negative correlation with other markets, making this ETF also a great security to optimize our portfolio.

### Conclusion
Instead of holding cash, this a solid, strategic short-term investment.

## Vanguard US Treasury 0-1 Year Bond UCITS ETF

Index: Bloomberg Short Treasury Index

Expense Ratio: 0.07%. Tracking Error Volatility: 0.0197%

### Overview
The Vanguard U.S. Treasury 0-1 Year Bond UCITS ETF invests in Government Bonds issued by United States. The bond maturities are between 0-1 years. The underlying bonds have AAA ratings. This is the highest possible credit-worthiness.

### Analysis
This is the cash-equivalent portion of our portfolio. The current macroeconomic scenario lies uncertainty ahead. As most of global investors have been doing in the past weeks, pouring capital in the money market is a too-attractive choice to not be considered. With the Fed Fund rate at 5.25%, the ETF offers an average yield of 4.5% and an average coupon of 1.25%, providing us with a secure, recurrent revenue stream while offering protection from the uncertain scenarios that are waiting ahead of us. Furthermore, the lingering concerns surrounding the US debt ceiling have exerted additional pressure on yields. We are excluding a US Government Debt default scenario; thus, this investment has only become more attractive.

### Conclusion
Overall, the Vanguard US Treasury 0-1 Year Bond ETF represents an attractive choice, particularly in an environment characterized by uncertainty, as it offers stability and potential returns.

## Invesco Bloomberg Commodity UCITS ETF

Index: Bloomberg Commodity Index

Expense Ratio: 0.19%. Tracking Error Volatility: 0.12%

### Overview
This fund aims to replicate the performance of the Bloomberg Commodity Index, which tracks prices of future contracts on physical commodities. The index itself is designed to minimize concentration in one sector and currently includes commodities futures in six sectors (energy, grains, industrial and precious metals, softs and livestock).

### Analysis
This ETF is well-balanced across various commodities. This makes it an appealing choice for investors looking to protect against inflation since commodity prices usually rise during inflationary periods. The ETF's most significant allocations are in the energy, agriculture, and metals sectors, offering a well-rounded blend of commodities that react differently to economic conditions. Energy commodities are influenced by geopolitical events and supply-demand factors, while agricultural commodities depend on weather conditions and global demand. Metals are impacted by industrial demand and macroeconomic elements.



### Conclusion
Despite our outlook on oil prices reported above and the recent underperformance, we deem a diversified exposure to commodities is still important to diversify asset classes, even though we reduced the overall weight.

## VanEck Green Metals ETF USD

Index: MVIS® Global Clean-Tech Metals Index

Expense Ratio: 0.59%. Tracking Error Volatility: -

### Overview
This ETF aims to track MVIS® Global Clean-Tech Metals Index, which tries to capture the performance of companies involved in the production, refining, processing and recycling of green materials essential for the ESG transition.

### Analysis
Because of the secular trend nurtured by the inevitable "green transition", we decided to keep this fund in our portfolio.

Indeed, it tracks the movement of companies that are involved in the production, refining, processing and recycling of green materials essential for the aforementioned "green transition". The choice was also reinforced by the continuing lack of an attractive ETF that directly tracks the evolution of the underlying commodities needed for the transition (issue partially offset by the global diversification of the companies tracked by this ETF).



### Conclusion
Due to the inevitability of the environmental transition we strongly think to be appropriately positioned to surf the green wave through this ETF.

## iShares 25+ Year Treasury STRIPS Bond ETF

Index: ICE BofA Long US Treasury Principal Strips Index

Expense Ratio: 0.1%. Tracking Error Volatility: 0.28%

### Overview
The investment seeks to track the investment results of the ICE® BofA® Long US Treasury STRIPS Index composed of the principal payments of U.S. Treasury bonds (specifically principal "STRIPS", also known as "Separate Trading of Registered Interest and Principal of Securities") with remaining maturities of at least 25 years

### Analysis
Treasury STRIPS are created when a bond's coupons are separated from the bond. The bond, minus its coupons, is then sold to an investor at a discount price. The difference between that price and the bond's face value at maturity is the investor's profit.
This ETF serves as the main hedge for the whole portfolio: the long-term zero-coupon bonds contained in the ETF will be greatly affected in the scenario of a recession, or a decrease in inflation data. In both cases, we expect to see lower yield curves worldwide and thus prices of these securities will react significantly.



### Conclusion
Including this ETF in our portfolio ensures more protection in case of an extremely severe recession. It is also appropriate to mention that last time the US almost defaulted on their debt, 30y and other long-term treasuries performed significantly well.

## Global X Lithium & Battery Tech ETF

Index: Solactive Global Lithium Index

Expense Ratio: 0.75%. Tracking Error Volatility: -

### Overview
The Global X Lithium & Battery Tech ETF (LIT) seeks to provide investment results that correspond generally to the price and yield performance, before fees and expenses, of the Solactive Global Lithium Index. The ETF invests in the full lithium cycle, from mining and refining the metal, through battery production.

### Analysis
This ETF invests in companies throughout the lithium cycle, including mining, refinement and battery production, cutting across traditional sector and geographic definitions. It provides exposure to companies producing lithium batteries, which are essential in the development of EVs, renewable energy storage, and mobile devices. While this is a riskier sector, we believe that current policies and the need for solutions to tackle the climate crisis will boost growth for this industry.



### Conclusion
The ETF has returned 2.76% YTD. While the performance is not disappointing, we believe that the US Inflation Reduction Act, which grants nearly $400bn to clean energy, will allow companies to invest heavily in the sector and grow significantly. Combining this with the substantial need to fight against climate change, we believe that the industry is destined to prosper in the future.

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# Quantitative Research Team

## Risk Report – May 2023

### Introduction

The main objective of this section is to assess and quantify the risk embedded in the Minerva IMS diversified passive selection fund built by the portfolio team. We use a daily perspective on the potential extreme behavior of a basket of assets selected by the portfolio analysts. The analysis will include three VaR and ES models (two parametric and one non-parametric) and the Black-Litterman optimization algorithm to inform the choice of component's weights.

As the Investment Risk division, our focus is the estimation of the two main risk indicators:

- The daily Value at Risk (VaR): the maximum portfolio loss that occurs with α% of probability over a time horizon of 1 day. For instance, if the VaR (α=5%) = -3.00%, it means that tomorrow there is a 5% probability of encountering a loss in the interval [-100%, -3.00%] potentially;

- The daily Expected Shortfall (ES): the expected return on the portfolio in the worst α% of cases. So, it is just a mean of the returns lower than the VaR.

A simple technique to estimate these two measure is based on a historical approach: given a time series of returns of a financial security, we can easily compute the desired quantile of the historical distribution to estimate the VaR, and, after that, estimate the ES just by averaging the values below this threshold.
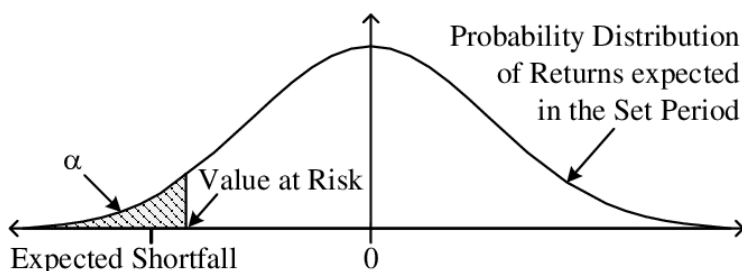
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However, this naive approach is not well suited for our purpose: in fact, by considering our portfolio as a single financial asset, we are losing all the information that comes from all the components; moreover, with this approach we are simply focusing on the past behavior of the fund, while our main goal is to retrieve a risk metric for the future possible trends.

In order to overcome these issues, we propose two alternative techniques that provides better risk estimates:

• Parametric approach (simple approach and time-series modelling approach)
• Bootstrapping

The first method is very well suited for understanding the main vulnerabilities in the portfolio composition, while with the second one it is possible to observe how the metrics varied in the past quarters.

All the analysis has been conducted with Python.

## Parametric approach

In this section we propose to analyze VaR and ES separately for each ETF included in the portfolio and then, to estimate the VaR and ES for the whole fund by taking into account the correlation between portfolio constituents.

Parametric approach is based on the assumption that returns of a financial security follow some theoretical distribution. Thus, VaR and ES can be expressed as an $\alpha$-percentile of the distribution. The crucial step to accurately estimate VaR and ES is to select the appropriate distribution of returns and estimate it's parameters.

It is possible to state that stock returns do not follow Gaussian distribution due to the presence of "fat tails": unexpected events might have a huge impact on the stock prices, so it is possible to observe extreme values more frequently than a Normal distribution would predict. For this reason, we assume that stock returns follow a Student-t distribution, thus, the parameters to be estimated are the mean $\mu$, volatility $\sigma$ and number of degrees of freedom $\nu$.

To obtain more valid and robust results, we proceed with two alternative parameter estimation approaches – (a) simple approach, and (b) time-series modelling approach.

### Simple approach

Under the simple approach, we estimate the above-mentioned parameters in the following way:

1. We assume that the mean historical daily return of each security are a good estimate for the expected future return. Thus, $\mu$ is estimated as a simple average of daily returns.

2. Volatility of returns $\sigma$ is calculated as a simple standard deviation of returns.

3. Number of degrees of freedom $\nu$ is selected in a way that it best approximates the empirical distribution of returns. In order to do that, we used the Kolmogorov-Smirnov statistic that, for a given empirical cumulative distribution function $F$ and a proposal $Fn$, is:

$$Dn = \sup x |(Fn - F)|$$

Ideally it should be equal to 0 for a perfect fit, so our goal is to minimize it by proposing different $\nu$ for Student-t distribution.

### Time-series modelling approach

Because the volatility of returns is not constant over time, it is often modelled by conditional heteroscedasticity processes. The most common way to model volatility is through a Generalized Autoregressive Conditional Heteroscedasticity model GARCH(p,q), where the forecast of the next-period volatility depends on the previous $p$ shocks to stock returns (derived from some mean model) and previous $q$ forecasts of volatility:

$$\sigma_{t+1|t}^2 = \omega + \sum_{i=1}^{p} \alpha_i \epsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j+1|t-j}^2$$

The advantage of GARCH model is that it allows to better estimate the current forecast of return volatility by putting more weight on more recent information. Thus, in the periods of market turbulence GARCH model will produce higher volatility forecasts than the simple average of squared deviations from the mean (see the graph at the bottom).

Because the portfolio is composed exclusively of passive instruments traded on liquid markets, we can assume that prices are efficient, and thus returns can be described by a constant mean model for GARCH(p,q) process, which implies that current mean estimates do not depend on previous returns or shocks. GARCH(p,q) then is estimated by Maximum Likelihood (MLE), which optimizes the distribution parameters. We subsequently use MLE estimates of distribution to derive VaR and ES.

## Parametric approach (continued)

### Value-at-risk

Once the parameters of stock returns are known, it is possible to calculate VaR. We estimate the VaR for 95% and 99% confidence level by applying the following formula:

$$VaR_\alpha = \sigma * T_\nu^{-1}(\alpha) + \mu$$

where $\sigma$ is the estimated volatility of a security, $T_\nu^{-1}(\alpha)$ is the $\alpha$-percentile of a Student-t distribution with $\nu$ degrees of freedom, and $\mu$ is the expected return of a stock.

### Expected shortfall

Expected shortfall is defined as a conditional expectation of loss, given that the loss occurred. If we introduce the assumption of a continuous distribution of returns of a security, then parametric expected shortfall is simply defined as a tail conditional expectation, and thus can in general be defined by the following formula for any security $X$:

$$ES_\alpha(X) = -\frac{1}{\alpha} \int_0^\alpha VaR_\gamma(X)\, d\gamma$$

Under the assumption of Student-t distribution with $\nu$ degrees of freedom it can be proven that the expected shortfall would be given as:

$$ES_\alpha(X) = \sigma * \frac{\nu + \left(T_\nu^{-1}(\alpha)\right)^2}{\nu - 1} \frac{\tau_\nu\left(T_\nu^{-1}(\alpha)\right)}{\alpha} + \mu$$

where $\sigma$ is the estimated volatility of a security, $T_\nu^{-1}(\alpha)$ is the $\alpha$-percentile of a Student-t distribution with $\nu$ degrees of freedom, $\tau_\nu(\cdot)$ is the probability density function of Student-t distribution with $\nu$ degrees of freedom and $\mu$ is the expected return of a stock.

We estimate the ES for 95% and 99% confidence level.

### Portfolio VaR and ES

Considering the correlation between the stocks, we estimate the VaR and ES of the whole portfolio for 95% and 99% confidence level by applying the following formulas:

$$VaR_{\alpha,ptf} \approx \sqrt{VaR_\alpha * \rho * VaR_\alpha'}$$
$$ES_{\alpha,ptf} \approx \sqrt{ES_\alpha * \rho * ES_\alpha'}$$

where $VaR_\alpha$ and $ES_\alpha$ are column vectors of individual stock VaR and ES, respectively and $\rho$ is the correlation matrix between securities

The approximation arises because of the assumption of Student-t distribution of returns – the formulas above become an equality the closer the distribution of returns is to the Gaussian.

### Results

GARCH results appear to be slightly higher than the simple approach ones. Indeed, while simple approach equally weights all observations, GARCH puts more weight on the most recent observations, thus, it better estimates the future volatility and allows to produce more reliable risk metrics.

|  | Simple approach | GARCH |
|---|---|---|
| VaR$_{95\%}$ | -0.96% | -1.22% |
| VaR$_{99\%}$ | -1.37% | -1.90% |
| ES$_{95\%}$ | -1.22% | -1.66% |
| ES$_{99\%}$ | -1.59% | -2.41% |

**Best & Worst Components GARCH**

|  | VaR 95 (GARCH) | VaR 99 (GARCH) | ES 95 (GARCH) | ES 99 (GARCH) |
|---|---|---|---|---|
| VDST.L | -0.03% | -0.05% | -0.05% | -0.07% |
| IE15.L | -0.43% | -0.76% | -0.66% | -1.09% |
| SCHP | -0.90% | -1.35% | -1.18% | -1.62% |
| AT1.L | -1.22% | -1.97% | -1.70% | -2.54% |
| VAPU.L | -2.18% | -3.36% | -2.92% | -4.15% |

|  | VaR 95 (GARCH) | VaR 99 (GARCH) | ES 95 (GARCH) | ES 99 (GARCH) |
|---|---|---|---|---|
| VDC | -3.56% | -6.45% | -5.49% | -9.20% |
| XMME.L | -3.91% | -6.54% | -5.63% | -8.77% |
| GMET | -4.37% | -6.43% | -5.65% | -7.61% |
| STUX.SW | -3.37% | -6.65% | -5.67% | -10.46% |
| LIT | -5.04% | -7.20% | -6.37% | -8.36% |

**Best & Worst Components Simple approach**

|  | VaR 95 | VaR 99 | ES 95 | ES 99 |
|---|---|---|---|---|
| VDST.L | -0.03% | -0.04% | -0.03% | -0.05% |
| IE15.L | -0.30% | -0.41% | -0.37% | -0.47% |
| SCHP | -0.89% | -1.25% | -1.11% | -1.44% |
| JT13.MI | -1.15% | -1.63% | -1.45% | -1.88% |
| AT1.L | -1.21% | -1.73% | -1.53% | -2.01% |

|  | VaR 95 | VaR 99 | ES 95 | ES 99 |
|---|---|---|---|---|
| STUX.SW | -2.30% | -3.28% | -2.90% | -3.79% |
| XMME.L | -2.68% | -3.75% | -3.34% | -4.31% |
| GOVZ | -2.93% | -4.10% | -3.65% | -4.72% |
| GMET | -3.45% | -4.93% | -4.36% | -5.70% |
| LIT | -4.28% | -6.24% | -5.50% | -7.37% |

## Bootstrapping

When estimating a certain metric, one of the main problems in Statistics is the lack of the whole population data and the consequent use of only a sample. In our case the population data is the complete historical price data of the securities that are part of our portfolio, in which we only have the data of recent years.

Bootstrapping is a statistical technique that by having only a sample of the population data, provides estimates of statistical metrics that are closer to the ones obtained from the population data.

Given a sample of size $n$, implementing bootstrap is very simple:

• Sample with replacement n times from the original sample (note that one observation could be selected more than once);
• Compute the metric of interest (in our case the VaR or ES) on this newly created sample and save it;
• Repeat the previous steps M times with M→+∞ (we have selected M=100.000 for instance);
• Average and compute the standard error of the metrics estimated in each step.

With this method, by estimating the expected shortfall and the standard errors, we can retrieve a more insightful view of our portfolio, but in this case, we are losing the risk contribution of each stock that we had in the previous case. Here, you can find the following estimates:

|  | Estimate | Standard error |
|---|---|---|
| $VaR_{95\%}$ | -1.07% | 0.12% |
| $VaR_{99\%}$ | -1.30% | 0.11% |
| $ES_{95\%}$ | -1.25% | 0.08% |
| $ES_{99\%}$ | -1.46% | 0.13% |

## Black –Litterman model

### Introduction

The Black-Litterman asset allocation model, created by Fischer Black and Robert Litterman, is a sophisticated portfolio construction method. The main trait that distinguishes the model is the Bayesian approach that is embodied in the inclusion of investors' expectations on future returns in building an optimal portfolio. Unlike the Markowitz optimization, in which return is maximized for a given level of risk, the Black-Litterman model combines the subjective views of an investor regarding the expected returns of one or more assets with the market equilibrium vector of expected returns to form a new estimate of expected returns. The resulting new vector of returns leads to intuitive portfolios with sensible portfolio weights.

### Inputs

To compute the portfolio composition, the model requires specific inputs. Some of them are common to other optimization models, like the expected excess returns and the variance-covariance matrix. In addition, we have:

• **VIEWS**: each investor has its own expectations about excess returns, which may deviate from the implied market ones. Views can be expressed in either absolute terms (Disney will have an absolute excess return of 5.25%) or in relative terms (Microsoft will outperform Apple by 2%). On the mathematic perspective, views are represented by a column vector with each element corresponds to a absolute/relative returns.

• **PICKING MATRIX**: this crucial element allows us to link each view to its corresponding asset. Mathematically, we have a matrix whose rows express the different views: absolute views have a single 1 in the column corresponding to the ticker's position, whereas relative views have positive numbers in the nominally outperforming asset columns and negative numbers in the nominally underperforming asset columns. All the other values are set to 0.

## Procedure

The Black-Litterman optimization process can be summarized in four parts:

- Estimate the (prior) implied expected returns using relative market capitalization weights and implied risk-aversion;
- Based on the investor views, build the view vector, the picking matrix and the (diagonal) matrix with the variance of each scenario;
- Use all of the previous inputs to compute the (posterior) "Black-Litterman" vector of expected excess returns;
- Use the vector of Black-Litterman posterior returns to compute the new weights for the portfolio.

## Key formulas and equations

The starting point is the computation of the implied excess returns via a reverse optimization method:

$$\Pi = \lambda \Sigma w_{market}$$

Where:
$\Pi$ is the Implied Excess Equilibrium Return Vector (N x 1 column vector),
$\Sigma$ represents the covariance matrix of excess returns (N x N matrix),
$\lambda$ is the risk aversion coefficient,
$w_{market}$ is the market capitalization weight.

The conversion from the prior return vector to the posterior Combined Return Vector ($E[R]$) is done according to:

$$E[R] = [(\tau\Sigma)^{-1} + P'\Omega^{-1}P]^{-1}[(\tau\Sigma)^{-1}\Pi + P'\Omega^{-1}Q]$$

Where:
$\tau$ is a scalar,
$P$ is is a matrix that identifies the assets involved in the views (K x N matrix),
$\Omega$ is a diagonal covariance matrix of error terms from the expressed views representing the uncertainty in each view (K x K matrix),
$Q$ is the View Vector (K x 1 column vector).
This formula can be intuitively interpreted as a weighted average between the (prior) implied returns and our views, with weights that depend on how much we are uncertain regarding every single view.