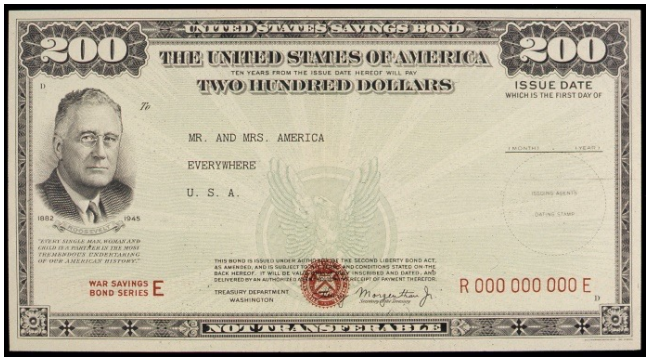
MIMS – Multi-Asset Global Opportunities Fund

Portfolio Management Team

Report – May 2023



Fund description

MIMS – Multi-Asset Global Opportunities Fund is an actively-managed fund by Minerva Investment Management Society, based on global macro opportunities as identified by the team and, possibly, ESG criteria.

The ultimate goal of this portfolio is to achieve long-term growth whilst controlling volatility. For this purpose, this fund will be comprised of a multitude of securities with the possibility, in exceptional cases, to take short term speculative positions. Hedging positions might be implemented through financial derivative instruments. To ensure diversification, this virtual portfolio is spread across geographies, sectors and asset classes, and is built through fundamental analysis, ESG integration and macroeconomic views.

In total, the asset allocation will aim to include around 30 different securities with a changing risky component to take advantage of contingent market conditions. The dynamic asset allocation prevents us from using a reference benchmark. The portfolio will be rebalanced every six months, with exceptional reviews to position for market shocks. The holdings only include instruments from the public markets, spread across equity, fixed income, real estate and commodities. ETPs might be considered to take additional exposures to niche markets.

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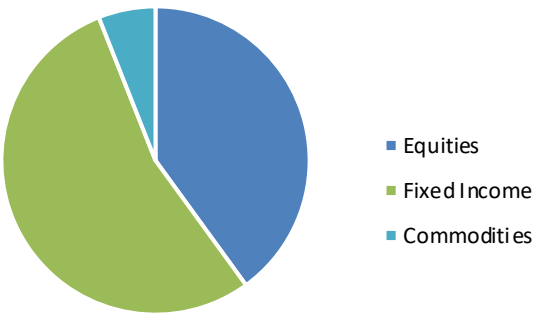
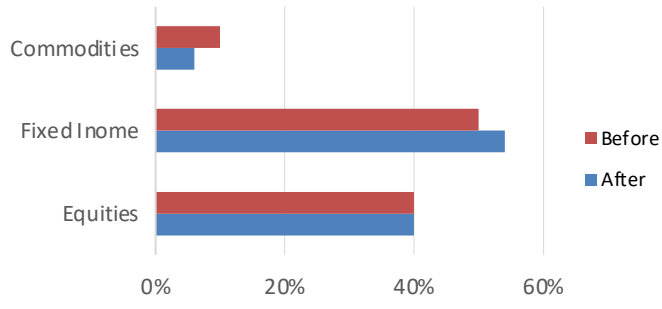
Asset Allocation



Asset Classes Rebalancing



Investment Approaches

Top-Down Approach

Starting from the macroeconomic outlook provided by the Strategy Research Team, the Investment Team identifies appealing industries, geographies and asset classes for which the best-performing securities will be analyzed thoroughly.
The Team applies a shared approach to the different asset classes by considering the main return drivers for each holding.

Bottom-Up Approach

If a security stands out to one of the Investment Analysts, the suggestion is discussed with the Team and additional analysis follows.
Long-term growth potential combined with attention to ESG standards and limited risk downsides both on a micro and macro level are required to consider the investment.

Research Contribution

The investment process uses internal research produced by the Research division of Minerva IMS.
The Strategy Team provides the outlook underlying the top-down approach. The Equity Team provides recommendations on potential stock holdings. Findings by both the Strategy and Macro Team are used for particular asset classes.

Performance

30.11.2022 – 28.04.2023

Update on Performance

The fund the current semester has reported a slight loss of **-4.37**%, beginning at **€ 9,747,589** and ending at **€ 9,321,577**.

This unexpected nominal negative return in EUR was mainly due to the **depreciation of USD** from 1.03 to 1.10. The exposure accounted for almost 40% of the portfolio. Considering the long-term nature of the fund and the high cost of EUR/USD hedging attributable to the difference in the policy rate, that portion of **the portfolio was unhedged**, and it was mainly the cause of the unexpected return.

Methodological Note

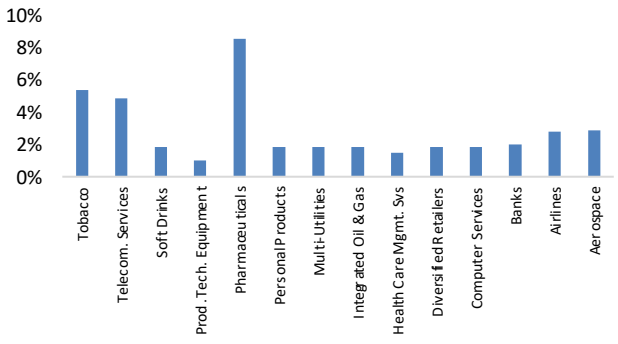
The performance starts by considering the final value of the previous allocation. Any security is held only in a discrete number, stock dividends and bond coupons are reinvested at the end of the day in which payments are received. The fund value is measured at the close of each trading day. Corporate events, dividend reinvestment and fund
rebalancing are carried out at the market close.

New Positioning - Top 10 Holdings

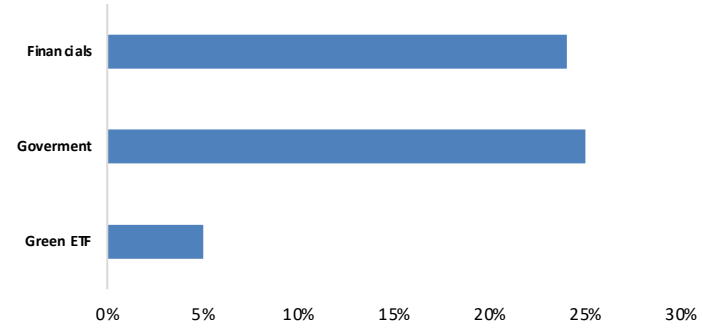
Security	Weight	Security	Weight
IUST PO STR 15-FEB-2053	10%	EUUNI 0.700 06-JUL-2051	5.0%
RBIV 8.750 22-NOV-2025 '24 MTN	5.0%	BEGV 0.650 22-JUN-2071	5.0%
HSBA 6.364 16-NOV-2032 FRN	5.0%	NOGV 3.000 14-MAR-2024	5.0%
ERST 6.693 14-NOV-2025 '24 MTN	5.0%	iShares € Corp Bond 0-3yr ESG UCITS ETF	5.0%
BLGG 2.375 30-APR-2026	5.0%	CRDI 5.850 15-NOV-2027 '26	4.0%

Sector Breakdown

Equity



Bond



New Entries	Outs
↗ UST PO STR 15-FEB-2053	↘ ISRAEL 2014 2 7/8% 29/01/24
↗ BLGG 2.375 30-APR-2026	↘ US TREASURY NOTE 2017 2 1/4% 31/01/24
↗ EUUNI 0.700 06-JUL-2051	↘ TREASURY GILT 2018 1% 22/04/24
↗ BEGV 0.650 22-JUN-2071	↘ BTP ITALY 2022 0% 29/11/23
↗ IMPERIAL BRANDS	↘ MERCK KGAA
↗ PFIZER	↘ PEPSICO
↗ BAYER	↘ NEXI S.p.A
↗ COCA-COLA	↘ NOVARTIS
↗ PHILIP MORRIS	↘ INTESA SANPAOLO S.p.A.
↗ BRITISH AMERICAN TOBACCO	↘ RYANAIR HOLDINGS
↗ UNITEDHEALTH GROUP	↘ Nickel (Commodity)
↗ AVIO S.P.A.	↘ 80% PUT SPX (Hedge)

RATIONALE FOR THE REBALANCING

According to the fund approach, every six months there is a rebalancing of the portfolio. We monitor in depth the evolution of the prices of the securities in our portfolio, when we believe the securities have reached their maximum potential, with the market price converging to our target, we substitute them with other securities with higher potential.

↘ OUTS

Beginning with the securities we removed, there are all the **short-term bonds**, which last semester were used as a cash substitute, considering that the long part of the curve was still unattractive. Regarding equities, we removed **Novartis, Ryanair and Intesa Sanpaolo** which we deem they reached their potential. We also removed **Pepsico** opting for Coca-Cola, which has better relative valuations.
Finally, we also decided not to roll over the **80% put option**, which was used to hedge the tail risk last semester, whose role has been ideally taken over by the introduction of ultra-long government bonds.

↗ NEW ENTRIES

The principal innovation this semester is the introduction of **ultra-long government bonds** as a hedge in our portfolio instead of the aforementioned put option. The rationale is that, in case major disruptions would occur in the global economy, central banks could reduce interest rates rapidly, increasing the market value of these types of bonds considering their long modified duration. We did not implement this in the past as the yields on the longer part of the curve were compressed. By contrast, now we deem them to be attractive. To reach this goal, we inserted a **US 2053 STRIP,** and **European Union** and **Belgium** long-term bonds, which also provide us with a palatable additional spread over the **German Bund**, for a relatively low additional risk.
Moving to the new entries on the equity side, we have several companies in the **tobacco** industry. Despite not being perfectly suitable under ESG criteria, we consider them to be strongly undervalued by the market. Moreover, they generate constant dividend cash flows and a value profile which is necessary, in our view, in this volatile scenario.
Pharmaceutical is the other industry in which we implemented significant changes. We decided to move more to US equities, which we consider at better valuations compared with European ones. Hence, we included **Pfizer** and **UnitedHealth Group** in our portfolio. Other minor changes in equities were made.

= UNCHANGED

The core part of the bond portfolio, including the financial senior, has remained unchanged. We still consider them a viable investment option with a YTM of around 7% with no excessive risk associated to it.
Moreover, the exposure to Norwegian Crown remained in our portfolio after the depreciation from the last semester, as we still consider it a good pick within the forex market.

EUUNI 0.700 06-JUL-2051

As mentioned before, we inserted in our portfolio different bonds with ultra-long duration. In particular, we consider this one among the best new picks for our strategy, as it is the longest duration bond issued by EU along with its low coupon (EU bonds are still not allowed to be stripped).

We believe EU bonds could become the new safe haven asset for an exposure to the EUR, being fully guaranteed by all the member states (up to certain proportion). This would result in a reduction in the spreads, which may converge in the long run to the yield of the German Bund.

Pfizer

Pfizer is a US pharmaceutical company, specialized in the research, production and marketing of drugs. Since the pandemic, it has grown significantly because it was one of the leading companies to supply a Covid-19 vaccine. Now, it is offsetting the decline in demand for these vaccines with its wide and various range of other drugs it offers. For instance, it has just received approval from the European Commission for the marketing of a drug for acute and episodic migraine (disease that affects one in ten Europeans). Moreover, it is also expanding its position in the cancer treatment segment with the recent acquisition of Seagen, a pioneering company in cancer care.

Therefore, its product diversification and strong position in the pharmaceutical sector put it in a favorable spot for a defensive portfolio strategy. Moreover, the CFO announced that the company will focus on boosting shareholders return, who recently benefit from a ROE of 32.61% and a dividend yield of 3.23% (2.5% growth rate).

Imperial Brands

Imperial Brands (IMB), previously Imperial Tobacco, is a UK-based multinational, active since 1901 in the tobacco industry and leader in the production of fine-cut tobacco and tobacco papers. Operating in over 160 countries and with a market cap above £17.5bn as of May 2023, it is the world's 4th largest international cigarette company by market share and its brand include Davidoff, West, Gauloises Blondes, Montecristo, Golden Virginia, Drum, and Rizla. Such a strong and diversified portfolio, coupled with the global distribution and trusted history of the company, constitute its competitive advantage which helped mitigate the impact of regulatory and consumer preference changes in different markets, as clear from the relatively low beta (0.81). Imperial Brands has been investing in next-generation products such as e-cigarettes, contributing toward a greater diversification and reach of the new customer.

Despite being undervalued due to difficulties in meeting ESG investing criteria, IMB has constantly delivered a strong balance sheet and P&L performance. It has been actively focusing on improving its efficiency by introducing a cost-cutting program aimed at realizing savings of £150m by the end of FY23 and improving net revenue growth by 1-2%. Additionally, in 2023 they adopted a new ESG framework fully integrated into vision, purpose, and behavior. Moreover, Imperial Brands shows a strong dividend track record, demonstrating a commitment to returning value to the shareholders, with a relatively high Dividend Yield of around 6.8% and a Payout Ratio exceeding 53%.

Avio

Avio is an Italian company operating in the aerospace sector, listed on the Milan stock exchange in the STAR segment. In the last six months, Avio showed a negative performance, mainly driven by the failure of the VV22 space mission.

The aerospace sector has become increasingly strategic for European countries. Avio received € 340mn in funds for R&D from NRRP last year and over € 285mn this year. The funds received will be invested in the coming months for the development of new innovative products, which will further strengthen Avio's market position and product range.

In carrying out its operations, Avio consumes a large amount of electricity, which comes mostly from gas. European gas prices have fallen to the lowest level since the start of the energy crisis. A factor to consider is the one related to the aerospace sector which, due to its own nature, is characterized by high risks deriving from the possibility of aerospace launches' failures with heavy repercussions on the stock market price, as happened a few months ago.

Coca-Cola

The Coca-Cola Company is an American multinational corporation, best known as the producer of Coca-Cola. The drink industry company also manufactures, sells, and markets other non-alcoholic beverage concentrates, syrups and alcoholic beverages.

Coca-Cola's share is characterized by stability and a relatively low beta. Over the last decades, the stock price was affected by just two large drops, corresponding to the 2008 crisis and the beginning of the Covid-19 pandemic outbreak.

Looking at the current P/E ratio (27.9x), Coca-Cola appears to be cheaper than its main competitor share, PepsiCo (40.6x), which has had a very positive performance over the past year.

The company ensures a good dividend yield, equal to approx. 2.9%.

UnitedHealth Group

UnitedHealth Group Incorporated (UNH) is an American multinational offering healthcare products and insurance services. As of 2022, it ranked 11th (2nd for the Healthcare sector) on the Fortune 500 list of the world's largest companies by consolidated revenue. With operations in over 130 countries, the group offers healthcare services through 2 subsidiaries: Optum, delivering healthcare services, and UnitedHealthcare, an insurance and managed care company, accounting for roughly 80% of the group revenues.

This diverse business segments allows UNH to operate in different niches of the profitable Healthcare industry, therefore providing a stable revenue stream, less susceptible to market fluctuations, as reflected in the company's low beta of 0.68.

The diversification, coupled with a consistent history of strong earnings and revenue growth (+12.71% in FY2022), exceeding expectations since 2009, allowed UnitedHealth Group to reach its prominent financial position in the industry, with a market capitalization of around $ 460bn as of May 2023. Moreover, the company has constantly reported solid balance sheets and cash flows, with a low debt-to-equity ratio of 0.76 as of 2022. This provides the company with the flexibility to invest in growth opportunities and return capital to shareholders through dividends (DPS at $ 6.40 and a 29.3% Payout Ratio). Furthermore, thanks to this positioning, the group benefits from strong economies of scale and network, enabling it to reduce expenses and invest heavily in technology and R&D, contributing to increasing its competitive advantage and positions for future growth.

BLGG 2.375 30-APR-2026

This bond is a senior non-preferred corporate bond issued by Bayerische Landesbank on November 23, 2022. It matures on November 21, 2025, and has a fixed coupon rate of 3.75%, while the yield is currently 4.5%.

We believe that including this bond in our portfolio is a worthwhile investment for several reasons. Firstly, it has a short duration, with a modified duration of 2.38, implying a relatively limited price sensitivity to interest rate changes. Moreover, a 4.5% yield for a bond that is rated Aa3 by Moody's is in our view an excellent deal, with an implied default risk of only 0.3%.

Bayerische Landesbank (BLB) bonds are considered safe for several reasons. BLB is a state-owned bank of the Bavarian state government in Germany, and it is considered one of the most stable and reliable banks in the country. BLB is subject to the supervision of BaFin, the German financial regulatory authority, which imposes strict prudential and transparency rules. In addition, BLB has a high rating from leading rating agencies such as Moody's, Standard & Poor's, and Fitch. Currently, BLB has an A1 rating from Moody's, A+ from Standard & Poor's, and A+ from Fitch, indicating that the bank is considered very reliable with a low probability of default. Finally, BLB bonds are guaranteed by the Bavarian state government, meaning that in case of financial difficulties of the bank, the government is obliged to intervene to ensure the repayment, offering greater security to investors.

Philip Morris

Philip Morris International is one of the world's leading manufacturers of cigarettes and tobacco products. Indeed, the company is behind a wide range of some of the most well-known premium, mid-price and low-price brands such as Marlboro, Chesterfield, L&M, Iqos and Heets.

Even if it is based in New York, it sells its products in 180 countries outside the US with FY22 net revenues divided into the following geographical distribution: 38.2% European Union, 11.7% Eastern Union, 12.3% Middle East & Africa, 13.8% South & Southeast Asia, 16.3% East Asia & Australia, 5.9% Americas.

The company's core business is combustible tobacco, which represented 67.9% of FY22 net revenues ($ 21.6bn), flanked by the smoke-free segment whose percentage grew from 29.7% in 2021 to 32.1% in 2022 ($ 10.2bn). Indeed, the company is transitioning its business towards smoke-free alternatives, for example the popular IQOS (developed in-house) and Zyn pouches (made by Swedish Match), which target to account for 50% of revenues by 2025.

The recent acquisition of Swedish Match, which is the largest producer in Scandinavia of snuff, chew bags, tobacco- and nicotine-free pouches and matches, was made in the perspective of this shift.

Detailed Holdings

UST PO STR 15-FEB-2053	10.00%
RBIV 8.750 22-NOV-2025 '24 MTN	5.00%
HSBA 6.364 16-NOV-2032 FRN	5.00%
ERST 6.693 14-NOV-2025 '24 MTN	5.00%
BLGG 2.375 30-APR-2026	5.00%
EUUNI 0.700 06-JUL-2051	5.00%
BEGV 0.650 22-JUN-2071	5.00%
NORWAY. KINGDOM OF (GOVERNMENT)	5.00%
ISHARES € CORP BOND 0-3YR ESG UCITS ETF	5.00%
CRDI 5.850 15-NOV-2027 '26 MTN	4.00%
IMPERIAL BRANDS	2.00%
PFIZER	2.00%
BAYER	2.00%
CRUDE OIL	2.00%
GOLD	2.00%
COCA-COLA	1.85%
PHILIP MORRIS	1.85%
IBM	1.85%
BROOKFIELD INFRASTRUCTURE PARTNERS	1.85%

TARGET	1.85%
UNILEVER	1.85%
GSK	1.85%
ENI	1.85%
AIRBUS	1.85%
VERIZON	1.85%
ROCHE	1.70%
SOUTHWEST AIRLINES	1.60%
BRITISH AMERICAN TOBACCO	1.50%
UNITEDHEALTH GROUP	1.50%
VODAFONE	1.50%
AT&T	1.50%
UNITED AIRLINES	1.20%
AVIO S.P.A.	1.00%
SANOFI	1.00%
ASML	1.00%
BANK OF CHINA	1.00%
ICBC	1.00%
COPPER	1.00%
SILVER	1.00%

Quantitative Research Team

Risk Report – May 2023

Introduction

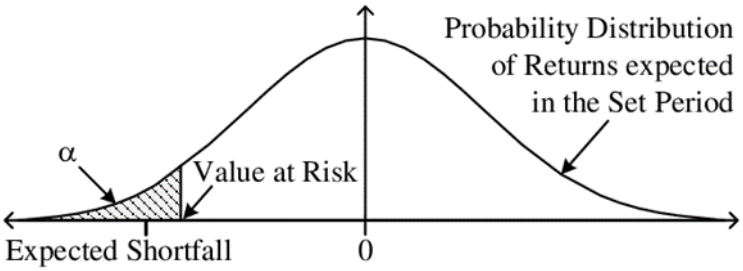
The main objective of this section is to assess and quantify the risk embedded in the Minerva IMS Multi Asset Global Opportunities Fund built by the portfolio team. We use a daily perspective on the potential extreme behavior of a basket of assets selected by the portfolio analysts. The analysis will include three VaR and ES models (two parametric and one non-parametric) and a Black-Litterman model for optimal allocation.

Our focus is the estimation of the two main risk indicators:

- The daily Value at Risk (VaR): the maximum portfolio loss that occurs with α% of probability over a time horizon of 1 day. For instance, if the VaR (α=5%) = -3.00%, it means that tomorrow there is a 5% probability of encountering a loss in the interval [-100%, -3.00%] potentially;

- The daily Expected Shortfall (ES): the expected return on the portfolio in the worst α% of cases. So, it is just a mean of the returns lower than the VaR.

A simple technique to estimate these two measure is based on a historical approach: given a time series of returns of a financial security, we can easily compute the desired quantile of the historical distribution to estimate the VaR, and, after that, estimate the ES just by averaging the values below this threshold.



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However, this naive approach is not well suited for our purpose: in fact, by considering our portfolio as a single financial asset, we are losing all the information that comes from all the components; moreover, with this approach we are simply focusing on the past behavior of the fund, while our main goal is to retrieve a risk metric for the future possible trends.

In order to overcome these issues, we propose two alternative techniques that provides better risk estimates:

- Parametric approach (simple approach and time-series modelling approach)
- Bootstrapping

The first method is very well suited for understanding the main vulnerabilities in the portfolio composition, while with the second one it is possible to observe how the metrics varied in the past quarters.

For both pieces of analysis we used daily market prices of portfolio constituents for the past six months. All the analysis has been conducted with Python.

Parametric approach

In this section we propose to analyze VaR and ES separately for each asset included in the portfolio and then, to estimate the VaR and ES for the whole fund by taking into account the correlation between portfolio constituents.

Parametric approach is based on the assumption that returns of a financial security follow some theoretical distribution. Thus, VaR and ES can be expressed as an α-percentile of the distribution. The crucial step to accurately estimate VaR and ES is to select the appropriate distribution of returns and estimate it's parameters.

It is possible to state that stock returns do not follow Gaussian distribution due to the presence of "fat tails": unexpected events might have a huge impact on the stock prices, so it is possible to observe extreme values more frequently than a Normal distribution would predict. For this reason, we assume that stock returns follow a Student-t distribution, thus, the parameters to be estimated are the mean μ, volatility σ and number of degrees of freedom ν.

To obtain more valid and robust results, we proceed with two alternative parameter estimation approaches – (a) simple approach, and (b) time-series modelling approach. For all parts of analysis, we use the last 100 return observations, which correspond almost to 4-months window.

Simple approach

Under the simple approach, we estimate the above-mentioned parameters in the following way:

1. We assume that the mean historical daily return of each security are a good estimate for the expected future return. Thus, μ is estimated as a simple average of daily returns.

2. Volatility of returns σ is calculated as a simple standard deviation of returns.

3. Number of degrees of freedom ν is selected in a way that it best approximates the empirical distribution of returns. In order to do that, we used the Kolmogorov-Smirnov statistic that, for a given empirical cumulative distribution function F and a proposal Fn, is:

$$Dn=\sup x|(Fn-F)|$$

Ideally it should be equal to 0 for a perfect fit, so our goal is to minimize it by proposing different ν for Student-t distribution.

Time-series modelling approach

Because the volatility of returns is not constant over time, it is often modelled by conditional heteroscedasticity processes. The most common way to model volatility is through a Generalized Autoregressive Conditional Heteroscedasticity model GARCH(p,q), where the forecast of the next-period volatility depends on the previous p shocks to stock returns (derived from some mean model) and previous q forecasts of volatility:

$$\sigma^2_{t+1|t} = \omega + \sum_{i=1}^{p} \alpha_i \epsilon^2_{t-i} + \sum_{j=1}^{q} \beta_j \sigma^2_{t-j+1|t-j}$$

The advantage of GARCH model is that it allows to better estimate the current forecast of return volatility by putting more weight on more recent information. Thus, in the periods of market turbulence GARCH model will produce higher volatility forecasts than the simple average of squared deviations from the mean (see the graph at the bottom).

Because the portfolio is composed exclusively of equity instruments traded on liquid markets, we can assume that prices are efficient, and thus returns can be described by a constant mean model for GARCH(p,q) process, which implies that current mean estimates do not depend on previous returns or shocks. GARCH(p,q) then is estimated by Maximum Likelihood (MLE), which optimizes the distribution parameters. We subsequently use MLE estimates of distribution to derive VaR and ES.

Parametric approach (continued)

Value-at-risk

Once the parameters of stock returns are known, it is possible to calculate VaR. We estimate the VaR for 95% and 99% confidence level by applying the following formula:

$$VaR_\alpha = \sigma * T_\nu^{-1}(\alpha) + \mu$$

where σ is the estimated volatility of a security, $T_\nu^{-1}(\alpha)$ is the α-percentile of a Student-t distribution with ν degrees of freedom, and μ is the expected return of a stock.

Expected shortfall

Expected shortfall is defined as a conditional expectation of loss, given that the loss occurred. If we introduce the assumption of a continuous distribution of returns of a security, then parametric expected shortfall is simply defined as a tail conditional expectation, and thus can in general be defined by the following formula for any security X :

$$ES_\alpha(X) = -\frac{1}{\alpha}\int_0^\alpha VaR_\gamma(X)\, d\gamma$$

Under the assumption of Student-t distribution with ν degrees of freedom it can be proven that the expected shortfall would be given as:

$$ES_\alpha(X) = \sigma * \frac{\nu + \left(T_\nu^{-1}(\alpha)\right)^2}{\nu - 1}\frac{\tau_\nu\left(T_\nu^{-1}(\alpha)\right)}{\alpha} + \mu$$

where σ is the estimated volatility of a security, $T_\nu^{-1}(\alpha)$ is the α-percentile of a Student-t distribution with ν degrees of freedom, $\tau_\nu(\cdot)$ is the probability density function of Student-t distribution with ν degrees of freedom and μ is the expected return of a stock.

We estimate the ES for 95% and 99% confidence level.

Portfolio VaR and ES

Considering the correlation between the stocks, we estimate the VaR and ES of the whole portfolio for 95% and 99% confidence level by applying the following formulas:

$$VaR_{\alpha,ptf} \approx \sqrt{\boldsymbol{VaR_\alpha} * \boldsymbol{\rho} * \boldsymbol{VaR_\alpha}'}$$
$$ES_{\alpha,ptf} \approx \sqrt{\boldsymbol{ES_\alpha} * \boldsymbol{\rho} * \boldsymbol{ES_\alpha}'}$$

where $\boldsymbol{VaR_\alpha}$ and $\boldsymbol{ES_\alpha}$ are column vectors of individual stock VaR and ES, respectively and $\boldsymbol{\rho}$ is the correlation matrix between securities

The approximation arises because of the assumption of Student-t distribution of returns – the formulas above become an equality the closer the distribution of returns is to the Gaussian.

Results

GARCH results appear to be more conservative than the simple approach ones. Indeed, while simple approach equally weights all observations, GARCH puts more weight on the most recent observations, thus, it better estimates the future volatility and allows to produce more reliable risk metrics.

	Simple approach	GARCH
$VaR_{95\%}$	-0.65%	-0.78%
$VaR_{99\%}$	-1.03%	-1.14%
$ES_{95\%}$	-0.90%	-1.01%
$ES_{99\%}$	-1.25%	-1.32%

TOP & BOTTOM 5 stocks (simple approach)

	VaR 95	VaR 99	ES 95	ES 99
3988.HK	-1.68%	-2.42%	-2.13%	-2.80%
KO	-1.69%	-2.42%	-2.14%	-2.80%
ULVR.L	-1.74%	-2.52%	-2.22%	-2.92%
IMB.L	-1.89%	-2.72%	-2.40%	-3.14%
ROG.SW	-1.91%	-2.74%	-2.42%	-3.19%

	VaR 95	VaR 99	ES 95	ES 99
AVIO.MI	-3.61%	-5.15%	-4.55%	-5.94%
TGT	-3.79%	-5.41%	-4.78%	-6.24%
LUV	-3.81%	-5.40%	-4.79%	-6.23%
ASML.AS	-4.24%	-6.27%	-5.50%	-7.41%
UAL	-4.93%	-7.19%	-6.33%	-8.41%

TOP & BOTTOM 5 stocks (GARCH)

	VaR 95 (GARCH)	VaR 99 (GARCH)	ES 95 (GARCH)	ES 99 (GARCH)
KO	-1.34%	-2.12%	-1.84%	-2.67%
PM	-1.98%	-3.19%	-2.75%	-4.07%
IBM	-2.07%	-3.42%	-2.94%	-4.46%
ROG.SW	-2.14%	-3.40%	-2.94%	-4.31%
IMB.L	-2.22%	-3.66%	-3.14%	-4.74%

	VaR 95 (GARCH)	VaR 99 (GARCH)	ES 95 (GARCH)	ES 99 (GARCH)
PFE	-4.08%	-6.45%	-5.59%	-8.15%
LUV	-4.04%	-6.50%	-5.61%	-8.32%
TGT	-4.45%	-7.08%	-6.12%	-8.95%
UAL	-4.60%	-7.13%	-6.19%	-8.76%
AVIO.MI	-4.50%	-9.02%	-7.69%	-14.43%

Bootstrapping

When estimating a certain metric, one of the main problems in Statistics is the lack of the whole population data and the consequent use of only a sample. In our case the population data is the complete historical price data of the securities that are part of our portfolio, in which we only have the data of recent years.
Bootstrapping is a statistical technique that by having only a sample of the population data, provides estimates of statistical metrics that are closer to the ones obtained from the population data.
Given a sample of size n, implementing bootstrap is very simple:

- Sample with replacement n times from the original sample (note that one observation could be selected more than once);
- Compute the metric of interest (in our case the VaR or ES) on this newly created sample and save it;
- Repeat the previous steps M times with M→+∞ (we have selected M=100.000 for instance);
- Average and compute the standard error of the metrics estimated in each step.

With this method, by estimating the expected shortfall and the standard errors, we can retrieve a more insightful view of our portfolio, but in this case, we are losing the risk contribution of each stock that we had in the previous case.

	Estimate	Standard error
$VaR_{95\%}$	-0.55%	0.19%
$VaR_{99\%}$	-0.95%	0.25%
$ES_{95\%}$	-0.95%	0.17%
$ES_{99\%}$	-1.23%	0. 26%

Black-Litterman weights

Stock	Weight
British American Tobacco	1.50
Imperial brands	2.00
UNITEDHEALTH GROUP	1.50
Avio S.p.A.	1.00
Coca-Cola	1.85
Pfizer	2.00
Philip Morris	1.85
Bayer	2.00
IBM	1.85
Brookfield Infrastructure Partners	1.85
target	1.85
Unilever	1.85
GSK	1.85
Sanofi	1.00
Roche	1.70
Eni	1.85
Vodafone	1.50
Airbus	1.85
ASML	1.00
AT&T	1.50
Verizon	1.85
Southwest airlines	1.60
United airlines	1.20
Bank of China	1.00
ICBC	1.00

Black –Litterman model

Introduction

The Black-Litterman asset allocation model, created by Fischer Black and Robert Litterman, is a sophisticated portfolio construction method. The main trait that distinguishes the model is the Bayesian approach that is embodied in the inclusion of investors' expectations on future returns in building an optimal portfolio. Unlike the Markowitz optimization, in which return is maximized for a given level of risk, the Black-Litterman model combines the subjective views of an investor regarding the expected returns of one or more assets with the market equilibrium vector of expected returns to form a new estimate of expected returns. The resulting new vector of returns leads to intuitive portfolios with sensible portfolio weights.

Inputs

To compute the portfolio composition, the model requires specific inputs. Some of them are common to other optimization models, like the expected excess returns and the variance-covariance matrix. In addition, we have:

- **VIEWS**: each investor has its own expectations about excess returns, which may deviate from the implied market ones. Views can be expressed in either absolute terms (Disney will have an absolute excess return of 5.25%) or in relative terms (Microsoft will outperform Apple by 2%). On the mathematic perspective, views are represented by a column vector with each element corresponds to a absolute/relative returns.

- **PICKING MATRIX**: this crucial element allows us to link each view to its corresponding asset. Mathematically, we have a matrix whose rows express the different views: absolute views have a single 1 in the column corresponding to the ticker's position, whereas relative views have positive numbers in the nominally outperforming asset columns and negative numbers in the nominally underperforming asset columns. All the other values are set to 0.

Procedure

The Black-Litterman optimization process can be summarized in four parts:

- Estimate the (prior) implied expected returns using relative market capitalization weights and implied risk-aversion;
- Based on the investor views, build the view vector, the picking matrix and the (diagonal) matrix with the variance of each scenario;
- Use all of the previous inputs to compute the (posterior) "Black-Litterman" vector of expected excess returns;
- Use the vector of Black-Litterman posterior returns to compute the new weights for the portfolio.

Key formulas and equations

The starting point is the computation of the implied excess returns via a reverse optimization method:

$$\boldsymbol{\Pi = \lambda \Sigma w_{market}}$$

Where:
Π is the Implied Excess Equilibrium Return Vector (N x 1 column vector),
Σ represents the covariance matrix of excess returns (N x N matrix),
λ is the risk aversion coefficient,
w_{market} is the market capitalization weight.

The conversion from the prior return vector to the posterior Combined Return Vector ($E[R]$) is done according to:

$$\mathbf{E[R] = [(\tau\Sigma)^{-1} + P'\Omega^{-1}P]^{-1}[(\tau\Sigma)^{-1}\Pi + P'\Omega^{-1}Q]}$$

Where:
τ is a scalar,
P is is a matrix that identifies the assets involved in the views (K x N matrix),
Ω is a diagonal covariance matrix of error terms from the expressed views representing the uncertainty in each view (K x K matrix),
Q is the View Vector (K x 1 column vector).
This formula can be intuitively interpreted as a weighted average between the (prior) implied returns and our views, with weights that depend on how much we are uncertain regarding every single view.