MIMS – Long-Short Equity Fund

Portfolio Management Team

Report – May 2023

Fund Description

MIMS – Long Short Equity Fund is a semi-automated actively-managed fund by Minerva Investment Management Society, based on a zero-net investment 'multi-factor' strategy. The Fund has the investment objective of achieving a positive absolute return throughout all market conditions, maintaining a constant euro exposure at each rebalancing.

Market Update

- During the last six months, the S&P 500 index and the Nasdaq Composite index gained respectively 2.83% and 10.24%. In Europe, the Euro STOXX 600 index raised by 5.61%. Japan's benchmark stock index, the Nikkei 225, gained 9.15% during the same period.

- Despite lingering concerns about a potential recession, central banks have chosen to continue to raise interest rates in the first half of 2023. The US Federal Reserve, for instance, raised rates three times by 25 basis points, resulting in the fed funds reaching the 5% - 5.25% level. The US CPI data for April showed a better-than-expected deceleration in inflation, with the headline inflation rate dropping to 4.9% from 6.5% registered in December. In Europe, April's CPI data showed a decline to 7% from December's 9.2%.

- On April 9, Kazuo Ueda was appointed as the new BOJ Governor. Investors expected a re-evaluation of the YCC policy, which has been in place since September 2016 to address low inflation but that also had a negative impact on Japan's sovereign bonds during the last year. At the first meeting scheduled for April 27-28, however, Ueda did not take any action despite the currently high level of inflation in Japan, going against market expectations.

- In March 2023, the banking sector faced major upheaval, with Silicon Valley Bank (SVB) and Credit Suisse (CS) collapsing within few days of each other. Moreover, the decision to write off CHF 16 bn ($17.5 bn) of CS's AT1 debt as part of a forced rescue merger with UBS caused significant disruption in financial markets and further financial instability. The aftermath of these events had far-reaching consequences, causing widespread concern and more deposits' withdrawals, with some market observers suggesting a halt in the tightening cycle. Lastly, on May 1st, JPMC acquired a portion of assets and deposits of First Republic Bank after its failure.

Head of Asset Management
Davide Giuliano: davide.giuliano@studbocconi.it

Head of Portfolio Management
Gianluca Caffi: gianluca.caffi@studbocconi.it

Portfolio Manager
Matteo Consalvo: matteo.consalvo@studbocconi.it

Portfolio Analysts
Pietro Cordero: pietro.cordero@studbocconi.it
David Overbeck: david.overbeck@studbocconi.it
Alberto Pavanello: alberto.pavanello@studbocconi.it
Matilde Zampa: matilde.zampa@studbocconi.it

Factor Investing Strategy	
VALUE	MOMENTUM
QUALITY	VOLATILITY
SIZE	ESG

3 Steps Investment Approach

Multi Factor Analysis

Fundamental metrics are identified that best proxy each of the 6 factors on which the investment style is grounded.
The process involves theoretical-based frameworks as well as empirical evaluations. Cross-team expertise and Minerva IMS insights are deployed.

Screening and Normalization

Stocks are evaluated on the basis of their exposure to each single factor. Outliers are substituted through a 3-step Winsorization procedure for every factor.
The output of the process is a synthetic score, which is then used to rank all the stocks.

Strategic Asset Allocation

Portfolio allocation comes to live. Based on the ranking produced, long and short positions are taken accordingly.
Macro environment is always monitored. Significant changes may lead to reconsider the chosen set of factors, or their weights, thus affecting the first step of the process.

Investment Approach

The Fund uses a 'multi-factor' based investment style adopting a quantitative proprietary model in order to achieve a systematic, rule-based approach to stock selection. Stocks are selected from the broad US Equity market (S&P 500 index) and the European Equity market (Euro STOXX 600 index).

A score is produced with reference to each considered style factor: (1) 'value' (stocks with lower price-to-book ratio and lower EV/EBITDA than peers); (2) 'size' (in terms of free-float market cap); (3) 'momentum' (investments with relatively strong recent performance); (4) 'quality' (as reflected by indicators such as ROE, ROE 5y growth, D/E and Earnings quality); (5) 'low idiosyncratic volatility'; (6) 'ESG' factor (as conveyed by the Thomson Reuters ESG Score). A systematic procedure is implemented to isolate and substitute the most extreme observations with reference to each single factor, considering the average value and the standard deviation of the characteristic in analysis for every sector. Each factor is given a specific weight in the process of building a final score for each stock. Sector-neutrality is partially considered: the model can in fact take larger long or short positions in certain sectors, but only within defined limits.

Tactical Decisions

In rebalancing the previous portfolio and building the new one, we adopted significant changes: **we reversed the sign of Size and eliminated the Indebtedness Sustainability Factor**, bringing the number of factors back to six. Moreover, **we completely reviewed the Quality factory**, with respect to both its composition (see page 6 for details) and the combined effect it has when mixed with Value (and also Size). Finally, **we allowed every factor to have its own weight**, in order to give more relevance to certain factors based on the expected macroeconomic environment.

From a Macro standpoint, we come from six months of moderate stock indices growth and of low volatility. For what concerns interest rate hikes, the worst seems to be over. On the other hand, markets are starting to look expensive, at least in the US, with the PE and the CAPE ratio both above 20. Also, even though inflation is coming down, we are still well above the 2% threshold and we don't know how much time it will take for it to return to normal levels: this combination of "higher for longer" interest rates and high inflation poses significant risks to the economy. Moreover, the Ukraine war is still going on and, last but not least, we still have to see if the banking crisis is over or if some more contagion will emerge.

Therefore, our factor allocation will be conservative for this semester. Its core is an equally weighted combination (20% each) of Value, Size and Quality, which allows us to buy small high-quality companies at a reasonable price. Then, we add the Low Volatility and ESG factors (15% and 10%) as a "safety" component, to ensure we buy stable ESG-friendly stocks. Lastly, following all the academic evidence supporting its combination with other factors, we add momentum (15%). We believe that this updated model will be able to choose stocks coherently with the challenging macroeconomic scenario we expect.

Fund Factors

Value Factor (Buy cheap stocks, Sell expensive stocks)

- Price-to-Book Value (P/BV): following the broad evidence provided by existing literature (e.g., Fama-French (1993)), we regard a high P/BV as a signal of relative overvaluation. We thus consider it as a selling indicator, since it shows that the company's equity is very expensive when compared to its underlying book value.
- EV/EBITDA: we regard a high EV/EBITDA as a selling signal, because it shows that the company is not able to generate a satisfactory level of EBITDA when compared to the value of the assets used to generate it.

Momentum Factor (Buy recently best-performing stocks, Sell worst-performing stocks)

- MOM: following the evidence provided by Jegadeesh and Titman (1993) and Asness (1994), we consider momentum, defined as the sum of the 12 monthly returns preceding the last one divided by 11, as a buy signal. In practice, we assume that the stocks that had a recent high average return will keep doing well in the future. In other words, we assume that the market will not invert its trend soon.

Quality Factor (Buy high-quality stocks, Sell low-quality stocks)

- Return on Equity (ROE): we consider a high ROE as a signal of high profitability and thus a buy signal. Specifically, we are assuming that company's profitability will remain stable in the future and will be a reliable driver of future increases in stock prices.
- 5y growth in ROE: to account for the growth of companies, we assess the earnings increase over the last five years relative to the equity's book value from five years ago. This allows us to reward companies that showed an increase in profitability while smoothing earnings by considering a 5-year window.
- Debt-to-Equity (D/E): for the safety dimension of our quality factor we consider the D/E ratio. A high D/E ratio indicates an excessive level of debt for the firm, representing a risk and also inflating ROE when earnings are positive.
- Earnings Quality: for safety we also use the earnings quality to measure how reliable a company's reported net income is by comparing it to its cash from operations.

Low Volatility Factor (Buy low volatility stocks, Sell high volatility stocks)

- Standard deviation: we deem a higher standard deviation to be a selling signal, since it reveals a riskier situation where returns are less stable, and, consequently, less predictable.

Size Factor (Buy small-caps, Sell large-caps)

- Free-Float Market Capitalization: a lower market cap is assumed to be a buy signal, since small cap stocks have historically shown relatively better performances than large cap stocks (see Banz (1981), Reinganum (1981) for empirical evidence in the academic literature).

ESG Factor (Buy high ESG scores' stocks, Sell low ESG scores)

- Refinitiv ESG Score: we assume a higher ESG score to be a positive signal, since it reveals more attention to sustainability within a firm. Several papers show that, in the long run, a higher ESG score allows sustainable companies to perform equally or even better than traditional ones, showing an improvement in risk-adjusted returns.

Portfolio Composition

The rebalancing of the long-short portfolio consists in buying stocks with the highest total score and short-selling stocks with the lowest, while liquidating all our previous positions. To make our portfolio construction more accurate, **from this semester onwards each factor will be assigned a weight.** This new factor-weighting scheme allows us to increase the exposure to factors where we have a strong conviction and reduce exposure to factors that we forecast to be less relevant in the following period.

The total score for each security is the sum of the final scores of the factors after having applied the Winsorization technique and the data normalization procedure weighted with its factor specific weight.

Finally, consistently with the previous rebalancing of the portfolio, a «semi» sector neutrality has been implemented. Indeed, a cap of 18% has been applied to all sectors in order to avoid excessive over- or under-exposure either in the short or in the long leg of our strategy without altering significantly the inherent philosophy of the model. We consider this to be an optimal compromise in the balancing of two opposite necessities.

It is important to stress that the above-mentioned procedure did not involve stock-picking of any kind. In fact, companies were substituted only for the «semi» sector neutrality feature.

Factor weights

Value (20%)	**Price-to-Book**	***10%***
	EV/EBITDA	***10%***
Size (20%)	**Free-Float Market Cap.**	**20%**
Momentum (15%)	**Asness (1994) Momentum**	**15%**
Quality (20%)	**Profitability: ROE**	***6,66%***
	Growth: ROE 5y growth	***6,66%***
	Safety: D/E	***3,33%***
	Safety: Earnings quality	***3,33%***
Low Volatility (15%)	**Standard deviation**	**15%**
ESG (10%)	**Refinitiv ESG Score**	**10%**

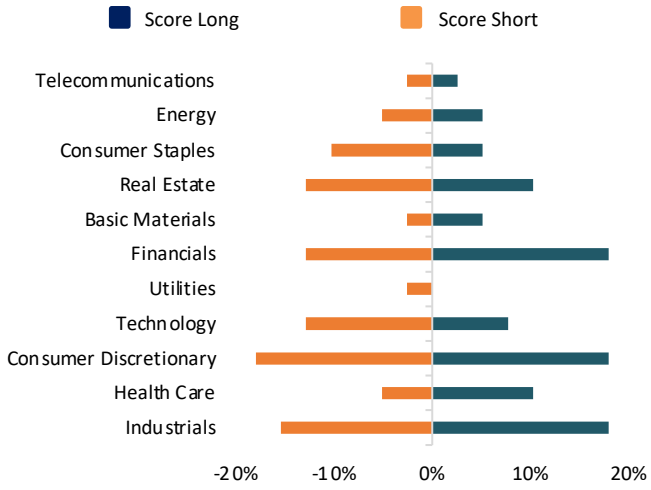
New Fund Positioning

S&P 500

ORACLE CORP	0.695	APPLE INC	-0.757
PRINCIPAL FINANCIAL	0.655	DISH NETWORK CORP	-0.759
MARATHON PETROLEUM	0.638	CHARLES SCHWAB CORP	-0.787
WW GRAINGER INC	0.633	MASTERCARD INC	-0.792
GILEAD SCIENCES INC	0.599	BERKSHIRE HATHAWAY INC	-0.799
OMNICOM GROUP INC	0.594	DEXCOM INC	-0.810
QUEST DIAGNOSTICS INC	0.590	AMERICAN TOWER CORP	-0.818
HARTFORD FINANCIAL INC	0.589	ENPHASE ENERGY INC	-0.849
WEYERHAEUSER CO	0.564	ILLUMINA INC	-0.895
PULTEGROUP INC	0.563	AMAZON.COM INC	-0.896
STEEL DYNAMICS INC	0.558	ETSY INC	-0.922
REGENCY CENTERS CORP	0.548	FIDELITY NATIONAL INC	-0.960
AON PLC	0.546	GENERAC HOLDINGS INC	-0.968
BORGWARNER INC	0.530	NVIDIA CORP	-0.982
REALTY INCOME CORP	0.527	PROLOGIS INC	-1.000
GLOBE LIFE INC	0.527	LINCOLN NATIONAL CORP	-1.028
STATE STREET CORP	0.526	NEXTERA ENERGY INC	-1.105
ULTA BEAUTY INC	0.518	TESLA INC	-1.123
HUMANA INC	0.518	NORWEGIAN CRUISE LTD	-1.349

Euro STOXX 600

AIB GROUP	0.748	FASTIGHETS BALDER B	-0.711
SERCO GROUP	0.733	FUTURE	-0.722
BALFOUR BEATTY	0.731	LATOUR INVESTMENT B	-0.725
VERALLIA	0.718	JUST EAT TAKEAWAY.COM	-0.767
ANDRITZ	0.679	SEGRO	-0.770
QIAGEN (XET)	0.673	NESTLE 'N'	-0.784
COCA-COLA HBC	0.665	GIVAUDAN 'N'	-0.797
BOSS (HUGO) (XET)	0.659	HERMES INTL.	-0.797
SOPRA STERIA GROUP	0.656	DIAGEO	-0.802
VIDRALA	0.652	NEL	-0.829
TECHNIP ENERGIES	0.620	SALMAR	-0.843
SAAB B	0.572	EMBRACER GROUP	-0.863
SOLVAY	0.563	ALLFUNDS GROUP	-0.874
TALANX AKTGSF. (XET)	0.562	SINCH	-0.877
HENKEL PREF. (XET)	0.531	ASML HOLDING	-0.883
TIETOEVRY	0.524	WISE A	-0.895
SODEXO	0.504	VONOVIA (XET)	-0.989
CARREFOUR	0.482	EQT	-1.081
TELEFONICA DTL. (XET)	0.479	OCADO GROUP	-1.281
SAFESTORE HOLDING	0.471	ADYEN	-1.585



Performance

The inception of the past portfolio took place on December 1, 2022. Therefore, the time frame considered goes from December 1, 2022, to April 28, 2022. Over the period, the portfolio obtained an absolute return of **€ 3,759** starting from **€ 100,000 of total exposure** (**9.46% annualized**) at the start date on the long and the short leg. If we consider the cumulative performance starting from November 21, 2021, the portfolio generated an absolute return of **€ 30,417**.

Keeping in mind that the net invested capital in this fund is zero euros, **our benchmark is to deliver positive returns**, an objective we achieved this semester. Moreover, it is worth noticing that in the same time frame of this report the S&P 500 and the STOXX 600 had a combined return (equally weighted portfolio) of 3.69%. We managed to make the same return, but without investing capital.

The final performance was also boosted by the short position on Signature Bank taken by the previous fund members, as well as by their bold choice to invert for one semester the sign of the size factor in order to penalise small caps, in contrast with what suggested by classical portfolio theory, to follow their views at the time.

In particular, over this semester the **S&P 500 long-short leg** performed well, as it contributed to the absolute return for an amount equal to **€ 1,254**. However, the **best performer** was the **Euro STOXX 600 long-short leg**, which produced gains for **€ 2,504**.

It can be noticed that the portfolio records a particularly negative performance during the first two months of the year 2023. However, such situations are an unavoidable consequence of keeping the same allocation for approximately six months, especially in **volatile periods** like the last one.

If we dive into the single components, we can see that the **best performers** in the S&P 500 leg of the portfolio were **Signature Bank** (short, -99.9% over the period), followed by **Truist Financial** (short, -32.3%) and **Norwegian Cruise** (short, -23.7%). The worst performers were instead **Charles Schwab** (long, -39.1%), **M&T Bank** (long, -28.4%) and **MetLife** (long, -23.5%). It is evident that the best performances were driven by short positions: for instance, the allocation benefitted from the failure of Signature Bank, which was shut down by federal regulators on March 12, 2023.

Looking at the **Euro STOXX 600 leg**, the **best performances** come from **Novo Nordisk** (long, +26.6% over the period), **Ocado Group** (short, -26.0%) and **Darktrace** (short, -23.4%). The worst performers were instead **Fortnox AB** (short, +39.3%), **Equinor** (long, -27.9%) and **Ericsson B** (long, -17.7%).

The positive performance achieved by our long-short portfolio over a time frame characterized by such a high level of uncertainty and low sentiment clearly shows the benefits provided by an appropriate combination of factors and highlights the **advantages of not building a long-only portfolio**.

Previous Allocation Performance (December 1, 2022 – April 28, 2023)

Source: Minerva Investment Management Society and Thomson Reuters Datastream. Past performance is not an indicator of future results.

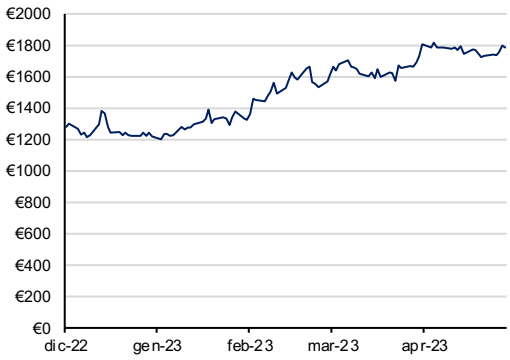
Top Performer: Signature Bank (Short)

Signature Bank was an American full-service commercial bank headquartered in New York City. Signature Bank **was shut down** by federal regulators on March 12, 2023. The bank's failure resulted from regulators' concern about **depositors withdrawing** large amounts of money after the failure of Silicon Valley Bank (SVB) and the fear of continued contagion.

Among the reasons why the model shorted Signature Bank there were: (i) **negative momentum**; (ii) **high volatility**; (iii) **low ROE**; (iv) **low size factor**.

Source: Refinitiv, Total Return Index

Worst Performer: Fortnox AB (Short)



Fortnox AB is a company based in Sweden which offers Internet-based business applications for companies, associations and accounting and auditing firms.

The model shorted the stock given (i) its **extremely high EV/EBITDA**: it seemed that the company was not able to generate a satisfactory level of EBITDA compared to the value of its assets; (ii) **high volatility**; (iii) **low ESG score**.

Source: Refinitiv, Total Return Index

Cumulative Performance (November 21, 2021 – April 28, 2023)

Source: Minerva Investment Management Society and Thomson Reuters Datastream. Past performance is not an indicator of future results.

Dissecting the quality factor

For the last five semesters, we have decided to include the value factor in our Long-Short Equity Fund. Unfortunately, though, the value factor does not take into account the quality of each stock. This is what brought us to do a deep dive on the Quality factor in order to better understand its composition and its interactions with the other factors.

Firstly, the construction of the Quality factor is very debated in the academic literature. The academic consensus of the most prominent papers, such as the one by Asness, Frazzini and Pedersen (2017), is that the quality factor should consist of characteristics that cover 3 dimensions: profitability, growth and safety.

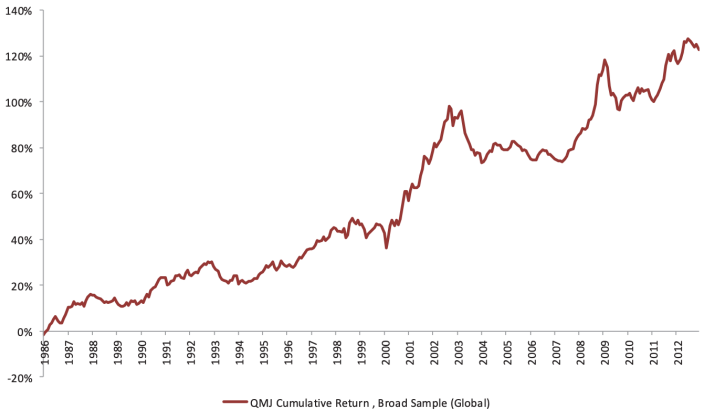
Their indication seems robust, as their Quality-minus-Junk (QMJ) factor delivers a positive return in 23 out of 24 countries examined and highly significant risk-adjusted returns across a broad and long sample. QMJ portfolios also demonstrate a negative exposure to the market, value, and size factors, while exhibiting positive alpha and relatively low residual risk. Additionally, QMJ shows a tendency to perform well during periods of market decline, which presents a challenge to a risk-based explanations of the positive returns of the factor but proves to be very useful as "insurance" for our portfolio[1].

Therefore, we follow the three authors' approach by constructing a price-agnostic quality factor that then can also be combined with a quality-agnostic value factor in order to get a combination called "Quality at a reasonable price (QARP)". Incidentally, Kalesnik and Kose (2016) show that QARP leads to a significant annualized alpha of 9.3%[2] while controlling for the four factors of the Carhart model.

As a result, our new quality factor will consist of (1) the ROE as our profitability metric, (2) the growth of the ROE over the last 5 years as our growth metric and (3) a combination of the company's D/E ratio and earnings quality as our safety metrics. The D/E ratio was selected in order to account for high ROE values due to excessive leverage, and earnings quality was chosen to counter "unhealthy" earnings growth. This represents in our view a coherent selection of characteristics from the several ones proposed for every dimension by Asness, Frazzini and Pedersen (2017).

Furthermore, in line with Hsu, Kalesnik and Kose (2017), we chose our safety metrics in a way to limit correlation with low volatility. A further idea could be to incorporate low volatility into the quality factor as the safety measure, but the empirical evidence by Blitz, van Vliet and Baltussen (2019), for instance, is strongly in favor of a single low volatility factor. The volatility factor is in fact the only factor which has generated a positive premium in every decade, and it is also the only factor which has delivered a solid premium over the most recent decade[3]. Moreover, by comparing the statistical significance of the factors' premia through their t-statistic, it can be seen that the volatility premium is the strongest factor[4]. Lastly, Blitz and van Vliet (2018) and van Vliet (2018) show that the low volatility factor works well with all the other factors and that the incorporation of other factors does not lead to a dilution of the low-risk exposure and to an increase in trading costs.

In conclusion, after this review of the quality factor, we are confident that the current combination of factors will be a solid foundation to tackle the challenging times that will unfold over the next semester.



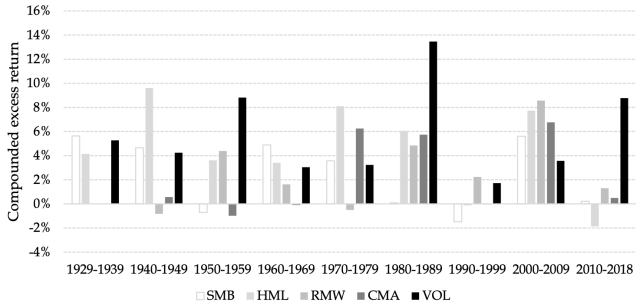
(1) Source: Quality Minus Junk (Asness, Frazzini and Pedersen, 2017)

Table 2. Long-Short Performance on Quality and Value Plus Quality. (U.S. Stocks, July 1963–January 2014)

Strategy	Average Ret (ann.)	Vol. (ann.)	t-stat	S.R.	Alpha (ann.)	t-stat	Market Exposure	Size Exposure	Value Exposure	Momentum Exposure
Simple Long Short	−0.4%	14.0%	0.29	−0.03	3.9%	2.52	−0.17	−0.77	−0.19	0.11
Diagonal Long Short	11.2%	24.6%	3.96	0.46	9.3%	3.10	−0.13	−0.40	1.15	0.16

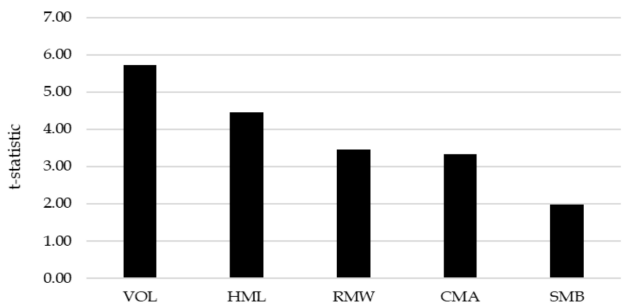
(2) Source: The Moneyball of Quality Investing (Kalesnik, Kose, 2016)

Exhibit 6: Factor premiums by decade



(3) Source: The Volatility Effect Revisited (Blitz, Vliet and Baltussen, 2019)

Exhibit 7: Strength of factor premiums, 1940-2018



(4) Source: The Volatility Effect Revisited (Blitz, Vliet and Baltussen, 2019)

Quantitative Research Team

Risk Report – May 2023

Introduction

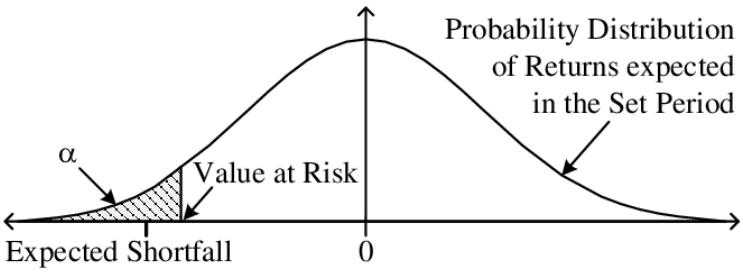
The main objective of this section is to assess and quantify the risk embedded in the Minerva IMS long-short equity fund built by the portfolio team. We use a daily perspective on the potential extreme behavior of a basket of assets selected by the portfolio analysts. The analysis will include three VaR and ES models (two parametric and one non-parametric) and an overview of how sentiment analysis can be considered a factor for short term investments.

As the Investment Risk division, our focus is the estimation of the two main risk indicators:

- The daily Value at Risk (VaR): the maximum portfolio loss that occurs with α% of probability over a time horizon of 1 day. For instance, if the VaR (α=5%) = -3.00%, it means that tomorrow there is a 5% probability of encountering a loss in the interval [-100%, -3.00%] potentially;

- The daily Expected Shortfall (ES): the expected return on the portfolio in the worst α% of cases. So, it is just a mean of the returns lower than the VaR.

A simple technique to estimate these two measure is based on a historical approach: given a time series of returns of a financial security, we can easily compute the desired quantile of the historical distribution to estimate the VaR, and, after that, estimate the ES just by averaging the values below this threshold.



Head of Quantitative Research
Emanuele Chiarini: +39 3478383128

Quant Analysts
Umberto Barbieri: +39 3332558188
Riccardo De Giovanni: +39 3398055958
Marco Folconaro: +39 347 329 2025
Giacomo Gregori: +39 347776 6335
Daniel Mitoiu: +40 745096655
Vasileios Stavropoulos: +30 697 802 6466
Erik Vik: +47 99156784
Francesco Zanettin: +39 3357257891

However, this naive approach is not well suited for our purpose: in fact, by considering our portfolio as a single financial asset, we are losing all the information that comes from all the components; moreover, with this approach we are simply focusing on the past behavior of the fund, while our main goal is to retrieve a risk metric for the future possible trends.

In order to overcome these issues, we propose two alternative techniques that provides better risk estimates:

- Parametric approach (simple approach and time-series modelling approach)
- Bootstrapping

The first method is very well suited for understanding the main vulnerabilities in the portfolio composition, while with the second one it is possible to observe how the metrics varied in the past quarters.

For both pieces of analysis we used daily market prices of portfolio constituents for the past 6 months,. All the analysis has been conducted with Python.

Parametric approach

In this section we propose to analyze VaR and ES separately for each asset included in the portfolio and then, to estimate the VaR and ES for the whole fund by taking into account the correlation between portfolio constituents.

Parametric approach is based on the assumption that returns of a financial security follow some theoretical distribution. Thus, VaR and ES can be expressed as an α-percentile of the distribution. The crucial step to accurately estimate VaR and ES is to select the appropriate distribution of returns and estimate it's parameters.

It is possible to state that stock returns do not follow Gaussian distribution due to the presence of "fat tails": unexpected events might have a huge impact on the stock prices, so it is possible to observe extreme values more frequently than a Normal distribution would predict. For this reason, we assume that stock returns follow a Student-t distribution, thus, the parameters to be estimated are the mean μ, volatility σ and number of degrees of freedom ν.

To obtain more valid and robust results, we proceed with two alternative parameter estimation approaches – (a) simple approach, and (b) time-series modelling approach. For all parts of analysis, we use the last 252 return observations, which correspond to 1-year window.

Simple approach

Under the simple approach, we estimate the above-mentioned parameters in the following way:

1. We assume that the mean historical daily return of each security are a good estimate for the expected future return. Thus, μ is estimated as a simple average of daily returns.

2. Volatility of returns σ is calculated as a simple standard deviation of returns.

3. Number of degrees of freedom ν is selected in a way that it best approximates the empirical distribution of returns. In order to do that, we used the Kolmogorov-Smirnov statistic that, for a given empirical cumulative distribution function F and a proposal Fn, is:

$$Dn = \sup x |(Fn - F)|$$

Ideally it should be equal to 0 for a perfect fit, so our goal is to minimize it by proposing different ν for Student-t distribution.

Time-series modelling approach

Because the volatility of returns is not constant over time, it is often modelled by conditional heteroscedasticity processes. The most common way to model volatility is through a Generalized Autoregressive Conditional Heteroscedasticity model GARCH(p,q), where the forecast of the next-period volatility depends on the previous p shocks to stock returns (derived from some mean model) and previous q forecasts of volatility:

$$\sigma^2_{t+1|t} = \omega + \sum_{i=1}^{p} \alpha_i \epsilon^2_{t-i} + \sum_{j=1}^{q} \beta_j \sigma^2_{t-j+1|t-j}$$

The advantage of GARCH model is that it allows to better estimate the current forecast of return volatility by putting more weight on more recent information. Thus, in the periods of market turbulence GARCH model will produce higher volatility forecasts than the simple average of squared deviations from the mean (see the graph at the bottom).

Because the portfolio is composed exclusively of equity instruments traded on liquid markets, we can assume that prices are efficient, and thus returns can be described by a constant mean model for GARCH(p,q) process, which implies that current mean estimates do not depend on previous returns or shocks. GARCH(p,q) then is estimated by Maximum Likelihood (MLE), which optimizes the distribution parameters. We subsequently use MLE estimates of distribution to derive VaR and ES.

Parametric approach (continued)

Value-at-risk

Once the parameters of stock returns are known, it is possible to calculate VaR. We estimate the VaR for 95% and 99% confidence level by applying the following formula:

$$VaR_\alpha = \sigma * T_\nu^{-1}(\alpha) + \mu$$

where σ is the estimated volatility of a security, $T_\nu^{-1}(\alpha)$ is the α-percentile of a Student-t distribution with ν degrees of freedom, and μ is the expected return of a stock.

Expected shortfall

Expected shortfall is defined as a conditional expectation of loss, given that the loss occurred. If we introduce the assumption of a continuous distribution of returns of a security, then parametric expected shortfall is simply defined as a tail conditional expectation, and thus can in general be defined by the following formula for any security X :

$$ES_\alpha(X) = -\frac{1}{\alpha}\int_0^\alpha VaR_\gamma(X)\, d\gamma$$

Under the assumption of Student-t distribution with ν degrees of freedom it can be proven that the expected shortfall would be given as:

$$ES_\alpha(X) = \sigma * \frac{\nu + \left(T_\nu^{-1}(\alpha)\right)^2}{\nu - 1}\frac{\tau_\nu\left(T_\nu^{-1}(\alpha)\right)}{\alpha} + \mu$$

where σ is the estimated volatility of a security, $T_\nu^{-1}(\alpha)$ is the α-percentile of a Student-t distribution with ν degrees of freedom, $\tau_\nu(\cdot)$ is the probability density function of Student-t distribution with ν degrees of freedom and μ is the expected return of a stock.

We estimate the ES for 95% and 99% confidence level.

Portfolio VaR and ES

Considering the correlation between the stocks, we estimate the VaR and ES of the whole portfolio for 95% and 99% confidence level by applying the following formulas:

$$VaR_{\alpha,ptf} \approx \sqrt{\boldsymbol{VaR_\alpha} * \boldsymbol{\rho} * \boldsymbol{VaR_\alpha}'}$$
$$ES_{\alpha,ptf} \approx \sqrt{\boldsymbol{ES_\alpha} * \boldsymbol{\rho} * \boldsymbol{ES_\alpha}'}$$

where $\boldsymbol{VaR_\alpha}$ and $\boldsymbol{ES_\alpha}$ are column vectors of individual stock VaR and ES, respectively and $\boldsymbol{\rho}$ is the correlation matrix between securities

The approximation arises because of the assumption of Student-t distribution of returns – the formulas above become an equality the closer the distribution of returns is to the Gaussian.

Results

GARCH results appear to be slightly higher than the simple approach ones, potentially due to the recent volatility in the markets. Indeed, GARCH puts more weight on the most recent observations, thus, it better estimates the future volatility and allows to produce more reliable risk metrics.

	Simple approach	GARCH
$VaR_{95\%}$	-1.89%	-2.05%
$VaR_{99\%}$	-2.69%	-3.41%
$ES_{95\%}$	-2.38%	-2.94%
$ES_{99\%}$	-3.10%	-4.59%

TOP & BOTTOM 5 stocks (GARCH)

	VaR 95 (GARCH)	VaR 99 (GARCH)	ES 95 (GARCH)	ES 99 (GARCH)
BRK-B	-1.54%	-2.27%	-1.99%	-2.67%
HIG	-1.94%	-2.85%	-2.50%	-3.33%
NESN.SW	-1.88%	-3.10%	-2.67%	-4.04%
O	-2.10%	-3.19%	-2.78%	-3.86%
HEN3.DE	-2.06%	-3.40%	-2.92%	-4.44%

	VaR 95 (GARCH)	VaR 99 (GARCH)	ES 95 (GARCH)	ES 99 (GARCH)
ALLFG.AS	-7.72%	-13.37%	-11.46%	-18.46%
ILMN	-7.68%	-13.62%	-11.66%	-19.30%
ENPH	-9.64%	-16.55%	-14.18%	-22.54%
SINCH.ST	-10.11%	-17.80%	-15.25%	-25.07%
DISH	-10.69%	-18.79%	-16.09%	-26.34%

TOP & BOTTOM 5 stocks (simple approach)

	VaR 95	VaR 99	ES 95	ES 99
NESN.SW	-1.72%	-2.46%	-2.18%	-2.84%
BRK-B	-1.99%	-2.86%	-2.53%	-3.30%
DGE.L	-2.08%	-2.98%	-2.63%	-3.44%
O	-2.14%	-3.07%	-2.71%	-3.56%
GILD	-2.30%	-3.29%	-2.91%	-3.80%

	VaR 95	VaR 99	ES 95	ES 99
NCLH	-7.02%	-10.00%	-8.85%	-11.53%
ENPH	-7.08%	-10.12%	-8.95%	-11.68%
OCDO.L	-7.22%	-10.24%	-9.08%	-11.80%
TKWY.AS	-7.64%	-10.80%	-9.58%	-12.42%
SINCH.ST	-9.32%	-13.20%	-11.70%	-15.18%

Bootstrapping

When estimating a certain metric, one of the main problems in Statistics is the lack of the whole population data and the consequent use of only a sample. In our case the population data is the complete historical price data of the securities that are part of our portfolio, in which we only have the data of recent years.

Bootstrapping is a statistical technique that by having only a sample of the population data, provides estimates of statistical metrics that are closer to the ones obtained from the population data.

Given a sample of size n, implementing bootstrap is very simple:

- Sample with replacement n times from the original sample (note that one observation could be selected more than once);
- Compute the metric of interest (in our case the VaR or ES) on this newly created sample and save it;
- Repeat the previous steps M times with $M \rightarrow +\infty$ (we have selected M=100.000 for instance);
- Average and compute the standard error of the metrics estimated in each step.

With this method, by estimating the expected shortfall and the standard errors, we can retrieve a more insightful view of our portfolio, but in this case, we are losing the risk contribution of each stock that we had in the previous case.

	Estimate	Standard error
$\mathbf{VaR_{95\%}}$	-1.69%	0.11%
$\mathbf{VaR_{99\%}}$	-2.87%	0.48%
$\mathbf{ES_{95\%}}$	-2.34%	0.23%
$\mathbf{ES_{99\%}}$	-3.57%	0.58%