MIMS – Long-Short Equity Fund

Portfolio Management Team

Report – December 2022

Fund Description

MIMS – Long Short Equity Fund is a semi-automated actively-managed fund by Minerva Investment Management Society, based on a zero-net investment 'multi-factor' strategy. The Fund has the investment objective of achieving a positive absolute return, through long-term capital appreciation.

Market Update

- **Central banks**, after aggressive monetary tightening implemented in 2022, are now wondering when it will come the time for a pivot, struggling in **balancing recession fears and inflation fight**. However, the pattern is not clear, and will be based on timely data as announced, for instance, by the ECB. The FED, after four consecutive 75bps rate hikes, turned to a smaller one (50bps) during the December FOMC meeting. Though an unexpectedly hawkish Powell signaled that it should not be considered as a pivot, **implicit rates are currently lower than those indicated** by the Fed officials in their dot plot. All the above leaves floor for **great uncertainty on the pivot's timing**.

- **China's Zero-Covid policy**, a major threat to supply chains in our globalized world, was finally eased in December. Unexpectedly, this may constitute an even greater threat in the short run. Indeed, the Chinese population is now facing an **incredible pandemic wave**. The Hang Seng climbed from November, but the Chinese government will have to promptly plan a strategy to restart the second largest economy limiting Covid effects that are frightening people worldwide.

- The main **Western indexes have been tumbling**, registering in 2022 their worst performances since the 2008 Financial Crisis. For instance, the S&P 500 lost 19% and the Nasdaq one third of its value. **We see recession concerns as the main issue in 2023.** Among others, a key role will be played by monetary policies, supply chain recovery, and energy shortage, as well as by the evolution of the War in Ukraine, which will turn one year long next February and is not expected to end soon.

- The **Bank of Japan** had been the only major central bank to keep expansive monetary stance. However, having finally experienced a significant jump in inflation, BoJ officials were recently forced to hike benchmark **interest rates to 0.5%**, also in order to sustain the yen, which has reached all-time lows against the dollar.

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Factor Investing Strategy	
VALUE	MOMENTUM
QUALITY	VOLATILITY
SIZE	ESG

3 Steps Investment Approach

Multi Factor Analysis

Fundamental metrics are identified that best proxy each of the 6 factors on which the investment style is grounded.
The process involves theoretical-based frameworks as well as empirical evaluations. Cross-team expertise and Minerva IMS insights are deployed.

Screening and Normalization

Stocks are evaluated on the basis of their exposure to each single factor. Through a 3-step Winsorization test, outliers are discarded with reference to each factor.
The output of the process is a synthetic score, on the basis of which stocks are ranked.

Strategic Asset Allocation

Portfolio allocation comes to live. Based on the ranking produced, long and short positions are taken accordingly.
Macro environment is always monitored. Significant changes may lead to reconsider the chosen set of factors, or their weights, thus affecting the first step of the process.

Investment Approach

The Fund uses a «multi-factor» based investment style adopting a quantitative proprietary model in order to achieve a systematic, rule-based approach to stock selection. Stocks are selected from the broad US Equity market (S&P 500 Index) and the European Equity market (Euro STOXX 600 Index).

A score is produced with reference to each considered style factor: (1) 'value' (stocks with lower price-to-book ratio and lower EV/EBITDA than peers); (2) 'momentum' (investments with relatively strong recent performance); (3) 'quality' (as reflected by indicators such as ROE and the difference between consensus forward and trailing EPS); (4) low idiosyncratic volatility; (5) size (in terms of market float); (6) ESG factor (as conveyed by Thomson Reuters ESG Score). A systematic procedure is implemented to isolate and discard the most extreme stocks with reference to each single factor. Each factor is given equal weight in the process of building a final score for each stock. Sector-neutrality is partially considered: the model can in fact take larger long or short positions in certain sectors, but only within defined limits.

Tactical Decisions

In rebalancing the previous portfolio and building the new one, we decided to maintain the foregoing set of factors, as the motivations that led us to select them in our view are still valid.

In addition, this semester we included in the score calculations the forward ND/EBITDA ratio. This was done mainly to take into account the changing macroeconomic scenario, with rising interest rates after more than a decade of low cost of borrowing.

We decided also to change the view on the size factor, exclusively for the next six months. In our new view, small firms are likely to be more affected than large caps by the combination of monetary tightening and economic slowdown.

Despite seeking sector-neutrality, we have a preference, and thus slightly overweight: Energy (due to the current geopolitical scenario), Financials (we believe interest rate hikes will be beneficial to banks without posing excessive risks related to non-performing loans, also in light of the overlays created during the pandemic crisis) and Utilities.

By constrast, we underweight consumer discretionary and technology (fears of deterioration of economic growth).

We believe this updated model to be able to choose stocks coherently with the macroeconomic scenario we expect, while maintaining a prudent defensive approach pending the FED's pivot.

Fund Factors

Value Factors (Buy cheap, Sell expensive)

- Price-to-Book Value (P/BV): following the broad evidence provided by existing literature (e.g., Fama-French (1993)), we regard a high P/BV as a signal of relative overvaluation. We thus consider it as a selling indicator, since it shows that the company's equity is very expensive if compared with its underlying book value.
- EV/EBITDA: we regard a high EV/EBITDA as a selling signal, because it shows that the company is not able to generate a satisfactory level of profits if compared to the value of the assets used to generate such profits.

Momentum Factor (Buy recently best-performing stocks, Sell worst-performing stocks)

- MOM: following the evidence provided by Jegadeesh and Titman (1993) and Asness (1994), we consider momentum, defined as the sum of the 12 monthly returns preceding the last one divided by 11, as a buy signal. In practice, we assume that the stocks that had a recent high average return will keep doing well in the future. In other words, we assume that the market will not invert its trend soon.

Quality Factors (Buy high-quality stocks, sell low-quality stocks)

- FW 12m EPS-Trailing EPS: a higher value of this metric represents a buy signal. Although not widespread, we introduced this factor in order to capture analysts' views (analysts' revisions) on the future of the company. It is indeed computed as the difference between the future twelve-months EPS forecasted by analysts and the trailing twelve-months recorded. We thus assume that a high positive value of this indicator will be associated with a stock price increase, as the stock price will mirror the future earnings' behavior.
- ROE: we consider a high ROE, normalized for industry influence, as a signal of high profitability, and, thus, a buy signal. Specifically, we are assuming that investors' profitability will maintain its trend in the future and will be a reliable driver of future increases in stock prices.

Volatility Factor (Buy low volatility, Sell high volatility)

- Standard deviation: we deem a higher standard deviation to be a selling signal, since it reveals a riskier situation where returns are less stable, and, consequently, less predictable.

Size Factor (Buy small-cap, Sell large-cap)

- The current macroeconomic landscape is characterized by high inflation, hawkish central banks, recessionary fears, and great uncertainty, due to, for instance, the war in Ukraine and the possibly ending zero-Covid policy in China. In such environment, we expect large caps to be more resilient and yield higher returns. Thus, we chose to short the size factor.

ESG Factor

- Thomson Reuters ESG Combined Score: we assume a higher ESG score to be a positive signal, since it reveals more attention to the sustainability of a firm. Although this factor has still few data recorded, market evidence suggests that, in the long run, a higher ESG score allows sustainable investments to perform equally or even better than traditional ones, showing an improvement in the long-term risk-adjusted returns.

Indebtness Sustainability Factor

- ND/EBITDA: considering the current economic environment, we decided to boost the scores of companies with a lower Debt-to-EBITDA ratio. The idea behind this choice is that, due to the rising interest rates, companies with a high reliance on financial leverage will be more likely to encounter difficulties in serving or refinancing their debt. In fact, the higher cost of borrowing will impact these firms' margins and the heavier dependence on external capital may negatively affect their financial stability and their performance.

Portfolio Composition

The rebalancing of the long-short portfolio consists in buying stocks with the highest total score and short-selling stocks with the lowest while liquidating all our previous positions.

The total score for each security is an equally-weighted average of the final factors' scores that each stock has registered, after having applied the Winsorization technique and the data normalization procedure.

The equally-weighted scheme has been adopted in order to preserve the identity of the factor. In this way, we avoided the possible drawbacks that optimization techniques, such as the ones based on the mean-variance approach, could have caused to our portfolio.

Finally, consistently with the previous rebalancing of the portfolio, a «semi» sector neutrality has been implemented. Indeed, a cap of 18% has been applied to all sectors in order to avoid excessive over- or under-exposure either in the short or in the long leg of our strategy without altering significantly the inherent philosophy of the model. We consider this to be an optimal compromise in the balancing of two opposite necessities.

It is important to stress that the above-mentioned procedure did not involve stock-picking of any kind. In fact, it was based on simple substitutions based on highest/lowest scores.

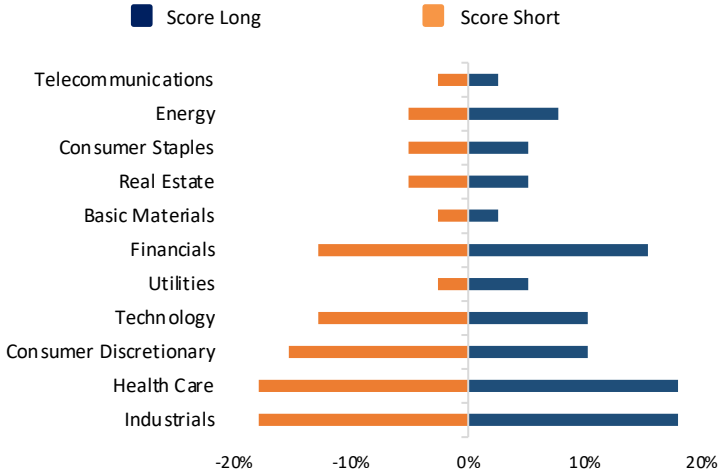
New Fund Positioning

S&P 500

CONSTELLATION ENERGY	1.145	MCCORMICK & COMPANY	-0.519
AMERISOURCEBERGEN	0.939	NORTHERN TRUST	-0.522
M&T BANK	0.674	ABIOMED	-0.516
LOCKHEED MARTIN	0.665	NXP SEMICONDUCTORS	-0.56
BROADCOM	0.661	CERIDIAN HCM HOLDING	-0.571
MCKESSON	0.659	TYLER TECHNOLOGIES	-0.574
MOLINA HEALTHCARE	0.652	ZEBRA TECHNOLOGIES 'A'	-0.576
HOWMET AEROSPACE	0.622	KINDER MORGAN	-0.579
ZIMMER BIOMET HDG.	0.600	BAXTER INTL.	-0.581
CARDINAL HEALTH	0.591	TRUIST FINANCIAL	-0.603
AMERIPRISE FINL.	0.578	NORWEGIAN CRUISE LINE	-0.606
METLIFE	0.563	CARMAX	-0.606
O REILLY AUTOMOTIVE	0.554	INTL.FLAVORS & FRAG.	-0.617
CBOE GLOBAL MARKETS	0.541	STANLEY BLACK & DECKER	-0.638
EOG RES.	0.539	NEWELL BRANDS (XSC)	-0.648
CHARLES SCHWAB	0.530	CINCINNATI FINL.	-0.663
CHIPOTLE MEXN.GRILL	0.522	WARNER BROS DISCOVERY	-0.739
DEVON ENERGY	0.510	SIGNATURE BANK	-0.761
PAYCOM SOFTWARE	0.471	AES	-0.927

EuroSTOXX 600

ERICSSON B	0.898	ALCON (SWX) ORD SHS	-0.547
EQUINOR	0.809	UNIBAIL RODAMCO WE	-0.588
NOVARTIS	0.772	NEL	-0.606
NOVO NORDISK 'B'	0.760	FORTNOX AB	-0.678
SPECTRIS	0.754	GRIFOLS ORD CL A	-0.713
L'OREAL	0.739	ALLREAL HOLDING	-0.785
HENNES & MAURITZ B	0.730	AMBU B	-0.792
ACS ACTIV.CONSTR.Y SERV.	0.718	VIRGIN MONEY UK	-0.829
ASML HOLDING	0.697	GETLINK	-0.894
ABB LTD N	0.687	UNIVERSAL MUSIC GROUP	-0.895
SKANSKA B	0.679	NEXI	-0.902
LONDONMETRIC PROPERTY	0.666	DARKTRACE	-0.905
GLAXOSMITHKLINE	0.662	ALFEN	-0.924
LLOYDS BANKING GROUP	0.595	BEIJER REF B	-0.927
ROCHE HOLDING	0.592	CONVATEC GROUP	-0.967
ANDRITZ	0.588	WISE A	-0.969
CASTELLUM	0.575	ADEVINTA	-1,053
SAGE GROUP	0.572	VANTAGE TOWERS N	-1,070
SSE	0.564	OXFORD NANOPORE TECH.	-1,080
RIO TINTO	0.551	OCADO GROUP	-1,513



Performance

The inception of the past portfolio took place on May 1, 2022. Therefore, the time frame considered goes from May 1, 2022, to November 30, 2022. Over the period, the portfolio obtained an absolute return of € **2,077.15** starting from **€ 100,000.00 of total exposition** (approx. **4% annualized**) at the start date.

If we consider the cumulative performance starting November 22, 2021, the portfolio generated a **total return above 25%** in the past year on a € 100,000 exposure at each start date, driven almost exclusively by the allocation of the first six months.

In particular, over this semester the **S&P500 long-short leg** did not performed well, as it contributed to the absolute return for an amount equal to **€ -100.4**. It follows that the overall performance was driven entirely by the **EuroSTOXX 600 leg** (**€ 2,177.6**). It can be easily noticed that the portfolio records a particularly **negative performance during the two rallies** that happened in the second part of the year (July and October), wiping out in both cases the good gains previously achieved. However, such situations are an unavoidable consequence of **rebalancing the portfolio only semesterly**, especially in **volatile periods** like this year.

If we dive into the single components, it can be easily noticed that the **best performers** in the S&P leg of the portfolio were **Constellation Energy** (long, +67.3% over the period), followed by **Marathon Petroleum** (long, +45.6%) and finally **Tesla** (short, -31%). Among the worst performers, Enphase Energy (short, +103.5% over the period), Boston Properties (long, -35.7%) and Las Vegas Sands (short, +35.3%).

Clearly, the allocation benefitted from the **overweight on the Energy sector**, even though the model was unable to properly select the stocks within the sector as Enphase Energy (short) performed significantly well. If we excluded Enphase Energy, also the S&P leg would show a positive performance.

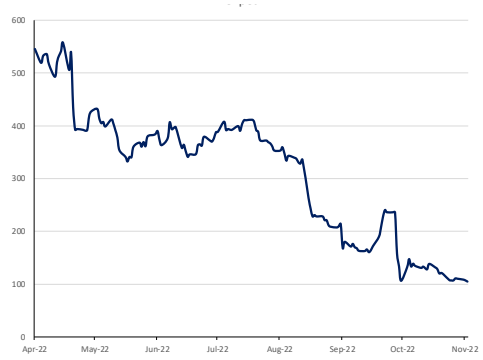
Looking at the **EuroSTOXX 600 leg**, the **best performances** come from **Orpea** (short, -80.8% over the period), **QT Group** (short, -46.3%) and **Ocado Group** (short, -34.5%). The worst performers were instead Wise (short, +59.1% over the period), BT Group (long, -31.2%) and Safestore Holdings (long, -29.4%).

The discrete performance achieved by our long-short portfolio over a time frame characterized by such an abnormal level of uncertainty and low sentiment clearly shows the benefits provided by an appropriate combination of factors and highlights the advantages of not building a long-only portfolio.

Previous Allocation Performance (May 1, 2022 – November 30, 2022)

Source: Minerva Investment Management Society and Thomson Reuters Datastream. Past performance is not an indicator of future results.

Top Performer: Orpea (Short)



Orpea is a French company active in nursing homes for aged people and mental health facilities, located mainly in Europe. In early 2022, it was **subject to company-specific scandals** related to the management of its facilities, which is the main reason for the **stock's negative performance**.

Among the reasons why the model shorted Orpea were: (i) extremely **negative momentum** factor due to the aforementioned scandals; (ii) particularly **high volatility**; (iii) low ROE. in particular, the **negative newsflow and sentiment** continued during the second part of 2022.

Total Return Index

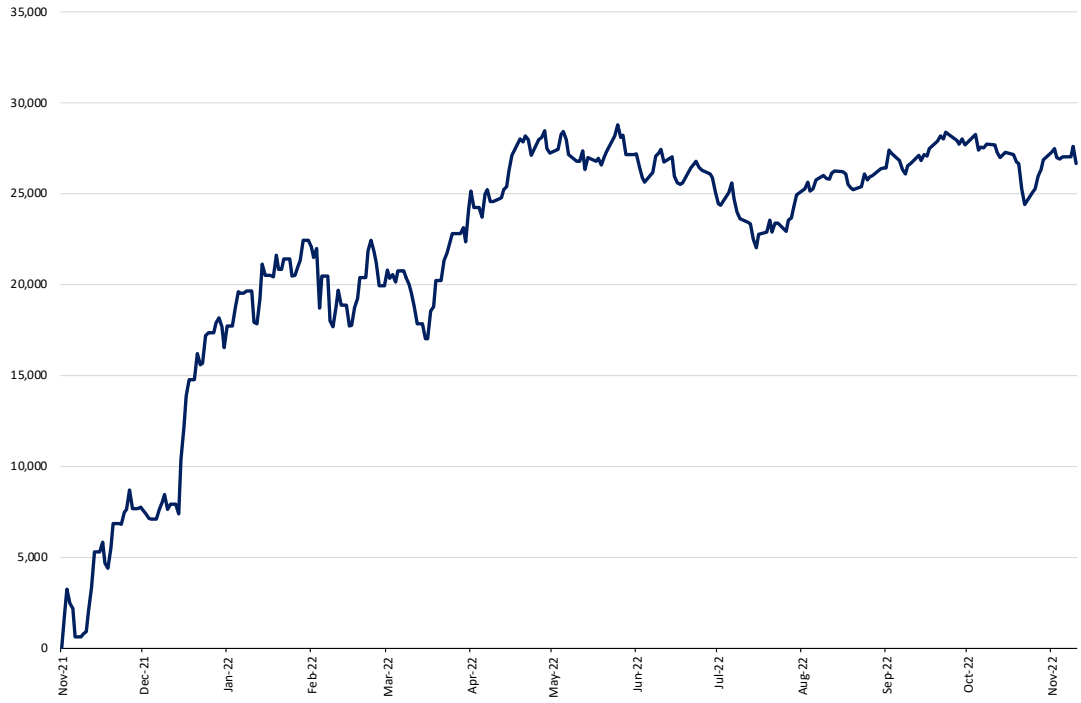
Worst Performer: Enphase Energy (Short)

Enphase Energy is a US-based company active in the field of renewable energy and the development of proprietary software for power generation management and monitoring.

The model shorted the stock given its **growth profile** (very high P/BV and EV/EBITDA, albeit with an equally high ROE) compared to the Energy sector, which typically trades at cheaper multiples. The failure to differentiate between "traditional" Energy and Renewables caused an excessive penalty for this stock. In addition, the very **favorable macro environment** for renewable producers exacerbated this error.

Total Return Index

Cumulative Performance (November 2021 – November 2022)



Source: Minerva Investment Management Society and Thomson Reuters Datastream. Past performance is not an indicator of future results.

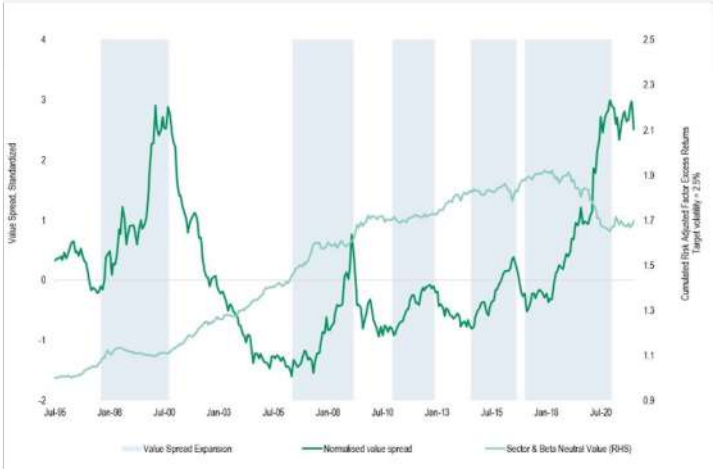
Value and Low Volatility: a Tried and True recipe

It is the fourth semester in a row that our analysts at the Long-Short Equity Fund have choose value instead of growth. So far, the choice has proven to be the right one.

As we navigate through times of high inflation, we found an interesting paper that supports our choice once again. **Dechow et al. (2021)** show that value stocks have low cash flow durations, causing them to underperform when discount rates decline. As interest rates are being hiked to fight high inflation, we believe that value stocks will outperform the market.

In addition, according to **Benoit Bellone** (Senior Quantitative Analyst in the Research Lab of the Quantitative Research Group at BNP Paribas Asset Management), in a recent article "Value Stocks Still Cheap Relative To Growth Sector Peers", the author writes that "Multi-factor equity strategies should prove more attractive for investors. The prospect of rising interest rates as central banks react to higher inflation further complicates the outlook for 'glamour' growth stocks.", which follows our belief that typical growth stocks such as FAANG are in for a potentially hard time, even though several of them have already suffered significant damage.

Several analysts have also been comparing the current market to the 2000 dot-com bubble, as depicted in the graph below.



Sources: BNP Paribas Asset Management

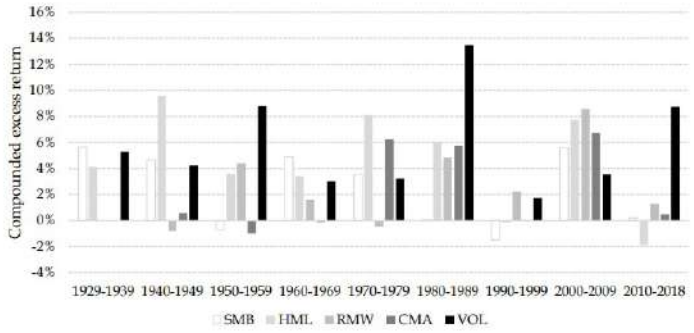
On another note, we also consider Low Volatility to be a strong indicator of positive future performance and that is why the analysts at the Long-Short Equity Fund are going with low volatility once again.

As demonstrated in a recent paper "The Volatility Effect Revisited" by **Blitz et al. (2019)**, the authors determine that "A low-risk approach has been effective for as far as the data goes back, across all major stock markets, from developed to emerging, within and across industries, across various market regimes and using different measures of risk.".

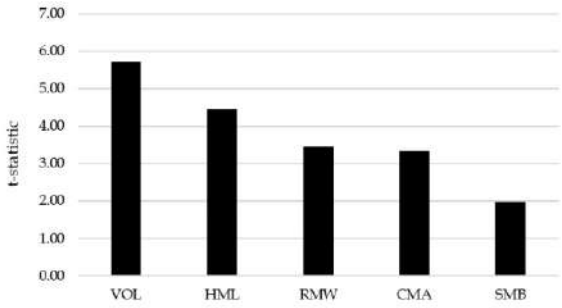
This recent study reviews a large number of markets and past performances, and it supports our conviction that low volatility drives strong performance.

The first graph below shows that volatility is one of the factors that most correlates with excess returns and it shows which factors were the most correlated with such over the past few decades, while the second graph shows the strength of each factor premium, from 1948 until 2018.

We are confident that this combination of factors is a solid foundation to tackle the challenging times that will unfold over the next semester.



Sources: Kenneth French data library, paradoxinvesting.com, Journal of Financial Economics website



Sources: Kenneth French data library, paradoxinvesting.com, Journal of Financial Economics website

Quantitative Research Team

Risk Report – December 2022

Introduction

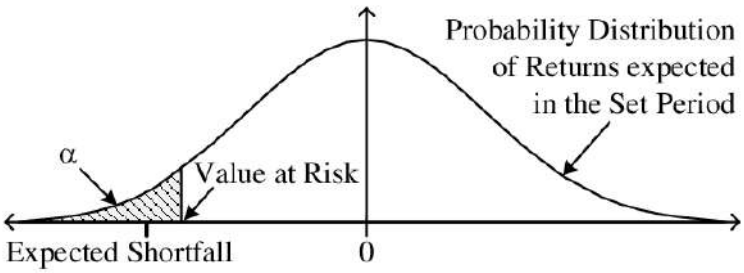
The main objective of this section is to assess and quantify the risk embedded in the Minerva IMS long-short equity fund built by the portfolio team. We use a daily perspective on the potential extreme behavior of a basket of assets selected by the portfolio analysts. The analysis will include three VaR and ES models (two parametric and one non-parametric) and an overview of how sentiment analysis can be considered a factor for short term investments.

As the Investment Risk division, our focus is the estimation of the two main risk indicators:

- The daily Value at Risk (VaR): the maximum portfolio loss that occurs with α% of probability over a time horizon of 1 day. For instance, if the VaR (α=5%) = -3.00%, it means that tomorrow there is a 5% probability of encountering a loss in the interval [-100%, -3.00%] potentially;

- The daily Expected Shortfall (ES): the expected return on the portfolio in the worst α% of cases. So, it is just a mean of the returns lower than the VaR.

A simple technique to estimate these two measure is based on a historical approach: given a time series of returns of a financial security, we can easily compute the desired quantile of the historical distribution to estimate the VaR, and, after that, estimate the ES just by averaging the values below this threshold.



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However, this naive approach is not well suited for our purpose: in fact, by considering our portfolio as a single financial asset, we are losing all the information that comes from all the components; moreover, with this approach we are simply focusing on the past behavior of the fund, while our main goal is to retrieve a risk metric for the future possible trends.

In order to overcome these issues, we propose two alternative techniques that provides better risk estimates:

- Parametric approach (simple approach and time-series modelling approach)
- Bootstrapping

The first method is very well suited for understanding the main vulnerabilities in the portfolio composition, while with the second one it is possible to observe how the metrics varied in the past quarters.

For both pieces of analysis, we used daily market prices of portfolio constituents for the past 6 months,. All the analysis has been conducted with Python.

Parametric approach

In this section we propose to analyze VaR and ES separately for each asset included in the portfolio and then, to estimate the VaR and ES for the whole fund by taking into account the correlation between portfolio constituents.

Parametric approach is based on the assumption that returns of a financial security follow some theoretical distribution. Thus, VaR and ES can be expressed as an α-percentile of the distribution. The crucial step to accurately estimate VaR and ES is to select the appropriate distribution of returns and estimate it's parameters.

It is possible to state that stock returns do not follow Gaussian distribution due to the presence of "fat tails": unexpected events might have a huge impact on the stock prices, so it is possible to observe extreme values more frequently than a Normal distribution would predict. For this reason, we assume that stock returns follow a Student-t distribution, thus, the parameters to be estimated are the mean μ, volatility σ and number of degrees of freedom ν.

To obtain more valid and robust results, we proceed with two alternative parameter estimation approaches – (a) simple approach, and (b) time-series modelling approach. For all parts of analysis, we use the last 252 return observations, which correspond to 1-year window.

Simple approach

Under the simple approach, we estimate the above-mentioned parameters in the following way:

1. We assume that the mean historical daily return of each security are a good estimate for the expected future return. Thus, μ is estimated as a simple average of daily returns.

2. Volatility of returns σ is calculated as a simple standard deviation of returns.

3. Number of degrees of freedom ν is selected in a way that it best approximates the empirical distribution of returns. In order to do that, we used the Kolmogorov-Smirnov statistic that, for a given empirical cumulative distribution function F and a proposal Fn, is:

$$Dn=\sup x|(Fn-F)|$$

Ideally it should be equal to 0 for a perfect fit, so our goal is to minimize it by proposing different ν for Student-t distribution.

Time-series modelling approach

Because the volatility of returns is not constant over time, it is often modelled by conditional heteroscedasticity processes. The most common way to model volatility is through a Generalized Autoregressive Conditional Heteroscedasticity model GARCH(p,q), where the forecast of the next-period volatility depends on the previous p shocks to stock returns (derived from some mean model) and previous q forecasts of volatility:

$$\sigma^2_{t+1|t} = \omega + \sum_{i=1}^{p} \alpha_i \epsilon^2_{t-i} + \sum_{j=1}^{q} \beta_j \sigma^2_{t-j+1|t-j}$$

The advantage of GARCH model is that it allows to better estimate the current forecast of return volatility by putting more weight on more recent information. Thus, in the periods of market turbulence GARCH model will produce higher volatility forecasts than the simple average of squared deviations from the mean (see the graph at the bottom).

Because the portfolio is composed exclusively of equity instruments traded on liquid markets, we can assume that prices are efficient, and thus returns can be described by a constant mean model for GARCH(p,q) process, which implies that current mean estimates do not depend on previous returns or shocks. GARCH(p,q) then is estimated by Maximum Likelihood (MLE), which optimizes the distribution parameters. We subsequently use MLE estimates of distribution to derive VaR and ES.

Parametric approach (continued)

Value-at-risk

Once the parameters of stock returns are known, it is possible to calculate VaR. We estimate the VaR for 95% and 99% confidence level by applying the following formula:

$$VaR_\alpha = \sigma * T_\nu^{-1}(\alpha) + \mu$$

where σ is the estimated volatility of a security, $T_\nu^{-1}(\alpha)$ is the α-percentile of a Student-t distribution with ν degrees of freedom, and μ is the expected return of a stock.

Expected shortfall

Expected shortfall is defined as a conditional expectation of loss, given that the loss occurred. If we introduce the assumption of a continuous distribution of returns of a security, then parametric expected shortfall is simply defined as a tail conditional expectation, and thus can in general be defined by the following formula for any security X :

$$ES_\alpha(X) = -\frac{1}{\alpha}\int_0^\alpha VaR_\gamma(X)\, d\gamma$$

Under the assumption of Student-t distribution with ν degrees of freedom it can be proven that the expected shortfall would be given as:

$$ES_\alpha(X) = \sigma * \frac{\nu + \left(T_\nu^{-1}(\alpha)\right)^2}{\nu - 1}\frac{\tau_\nu\left(T_\nu^{-1}(\alpha)\right)}{\alpha} + \mu$$

where σ is the estimated volatility of a security, $T_\nu^{-1}(\alpha)$ is the α-percentile of a Student-t distribution with ν degrees of freedom, $\tau_\nu(\cdot)$ is the probability density function of Student-t distribution with ν degrees of freedom and μ is the expected return of a stock.

We estimate the ES for 95% and 99% confidence level.

Portfolio VaR and ES

Considering the correlation between the stocks, we estimate the VaR and ES of the whole portfolio for 95% and 99% confidence level by applying the following formulas:

$$VaR_{\alpha,ptf} \approx \sqrt{\boldsymbol{VaR_\alpha} * \boldsymbol{\rho} * \boldsymbol{VaR_\alpha}'}$$
$$ES_{\alpha,ptf} \approx \sqrt{\boldsymbol{ES_\alpha} * \boldsymbol{\rho} * \boldsymbol{ES_\alpha}'}$$

where $\boldsymbol{VaR_\alpha}$ and $\boldsymbol{ES_\alpha}$ are column vectors of individual stock VaR and ES, respectively and $\boldsymbol{\rho}$ is the correlation matrix between securities

The approximation arises because of the assumption of Student-t distribution of returns – the formulas above become an equality the closer the distribution of returns is to the Gaussian.

Results

GARCH results appear to be slightly higher than the simple approach ones, potentially due to the recent volatility in the markets. Indeed, GARCH puts more weight on the most recent observations, thus, it better estimates the future volatility and allows to produce more reliable risk metrics.

	Simple approach	GARCH
$VaR_{95\%}$	-1.50%	-1.79%
$VaR_{99\%}$	-2.15%	-3.11%
$ES_{95\%}$	-1.90%	-2.69%
$ES_{99\%}$	-2.48%	-4.56%

TOP & BOTTOM 5 stocks (simple approach)

	VaR 95	VaR 99	ES 95	ES 99
NOVARTIS 'R'	-1.75%	-2.57%	-2.25%	-3.01%
ALLREAL HOLDING	-2.24%	-3.16%	-2.81%	-3.63%
MCKESSON	-2.36%	-3.46%	-3.03%	-4.02%
AMERISOURCEBERGEN	-2.37%	-3.50%	-3.07%	-4.09%
GETLINK	-2.44%	-3.50%	-3.09%	-4.05%

	VaR 95	VaR 99	ES 95	ES 99
NEL	-7.56%	-10.85%	-9.58%	-12.53%
ADEVINTA	-7.67%	-10.90%	-9.65%	-12.55%
NORWEGIAN CRUISE LINE HDG.	-7.95%	-11.43%	-10.09%	-13.25%
DARKTRACE	-8.21%	-11.76%	-10.40%	-13.63%
OCADO GROUP	-8.96%	-12.69%	-11.25%	-14.61%

TOP & BOTTOM 5 stocks (GARCH)

	VaR 95 (GARCH)	VaR 99 (GARCH)	ES 95 (GARCH)	ES 99 (GARCH)
NOVARTIS 'R'	-1.75%	-2.68%	-2.33%	-3.24%
ABB LTD N	-2.17%	-3.23%	-2.83%	-3.84%
ALLREAL HOLDING	-2.19%	-3.34%	-2.91%	-4.08%
METLIFE	-2.30%	-3.38%	-2.97%	-3.96%
ACS ACTIV.CONSTR.Y SERV.	-2.33%	-3.56%	-3.09%	-4.29%

	VaR 95 (GARCH)	VaR 99 (GARCH)	ES 95 (GARCH)	ES 99 (GARCH)
ALFEN	-7.43%	-14.07%	-11.97%	-21.08%
ABIOMED	-7.81%	-14.81%	-12.62%	-22.35%
OCADO GROUP	-9.71%	-16.17%	-13.93%	-21.60%
DARKTRACE	-12.57%	-25.55%	-21.86%	-41.80%
VANTAGE TOWERS N (XET)	-18.32%	-42.83%	-37.72%	-84.44%

Bootstrapping

When estimating a certain metric, one of the main problems in Statistics is the lack of the whole population data and the consequent use of only a sample. In our case the population data is the complete historical price data of the securities that are part of our portfolio, in which we only have the data of recent years.

Bootstrapping is a statistical technique that by having only a sample of the population data, provides estimates of statistical metrics that are closer to the ones obtained from the population data.

Given a sample of size n, implementing bootstrap is very simple:

- Sample with replacement n times from the original sample (note that one observation could be selected more than once);
- Compute the metric of interest (in our case the VaR or ES) on this newly created sample and save it;
- Repeat the previous steps M times with $M \rightarrow +\infty$ (we have selected M=100.000 for instance);
- Average and compute the standard error of the metrics estimated in each step.

With this method, by estimating the expected shortfall and the standard errors, we can retrieve a more insightful view of our portfolio, but in this case, we are losing the risk contribution of each stock that we had in the previous case.

	Estimate	Standard error
$VaR_{95\%}$	-1.42%	0. 14%
$VaR_{99\%}$	-2.39%	0. 51%
$ES_{95\%}$	-2.00%	0.25%
$ES_{99\%}$	-2.90%	0.42%