MIMS – Credit Default Swaps

Markets and Alternatives Research Team

Report – December 2021

Credit Default Swaps are credit or entity. derivatives that offer insurance in case of default of an underlying bond

In this report a correlation analysis of CDS spread and the main asset classes is conducted. Furthermore, a discrete time variant of the Hull pricing model is implemented to obtain the historical default probabilities implied in the market prices of the CDS on four countries debt: Greece, Italy, Spain, and France.

The historical data is then implemented to build a forecast model based on a Random Walk model. For each country we applied a Monte Carlo simulation to default probabilities and risk-free rates, obtaining a future distribution of the CDS spread.

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DEFINITION

Overview

A credit default swap (CDS) is a type of credit derivative that insurance to the CDS buyer in the case of a credit event of a reference entity, usually the corporation issuing a bond. The reference obligation refers to the debt instrument that the CDS is covering, such as a corporate bond.

The CDS buyer makes periodic premium payments to the CDS seller if the reference entity does not experience a credit event that prevents it from upholding its obligations to the CDS buyer.

In the case of a credit event, the CDS seller will have to provide compensation to the CDS buyer, constituting the notional amount covered, while the buyer will have to provide the insured financial instrument.

CDS Pricing

There are two main ways of displaying the price of a CDS:

CDS spread

Due to standardization, the most common CDS' pay a fixed coupon of 5% of the notional value for high-yield companies and 1% for investment-grade companies. Premium payments are usually made quarterly. As the actual value of the protection may differ from the standardized ones, CDS are often quoted with a different coupon rate called spread.

Upfront payment

If the credit spread differs from the standard coupon rate, an upfront payment may be involved.

It can be calculated as the difference between the present value of the credit spread and the present value of the fixed coupon, and therefore might be either positive (the protection buyer will make an upfront payment) or negative (the protection seller will make an upfront payment).

This is to account for the fact that credit risk differs amongst debt instruments of the same investment grade.

Types of CDS

Credit-default swaps can be categorized in different ways, the following included:

- **Single-name CDS**: issued for a single particular borrower. If there is a physical settlement, the cheapest-to-deliver method is used to determine the payoff, meaning by choosing the bond with the minimum difference between the spot price and futures price or the highest returns/implied repo rate.
- **Index CDS**: includes multiple reference entities. The main differences with a single-name CDS include the continuation of the CDS contract in the case of a credit event of one reference entity, which is removed from the index, followed by settling the removed entity as a single-name CDS or re-adjusting the premium payments and contract conditions. Index CDS can be further divided into funded or unfunded CDS, determining whether the CDS buyer is exposed to counterparty risk.
- **Tranche CDS**: constitute standardized collateralized debt obligations that cover only up until a specific segment of the loss distribution of underlying CDS indexes. For example, equity CDS tranches may cover up to 3% of the index in the case of losses from default.

Credit Events

The CDS contract will be triggered at the occurrence of a credit event, where the CDS seller will be expected to make the payoff. Credit events are defined in the contract, often depending on the type of CDS that is purchased.

In general, credit events include the following default occurrences:

- Bankruptcy: the reference entity declares bankruptcy and files for relief.
- Missed payments: the reference entity fails to make principal or interest payments within specified deadlines.
- Debt restructuring: the debt obligations of the reference entity are modified

CDS Settlement

In case of a credit event a cash or physical settlement is due to the CDS buyer.

- Cash settlement: cash is transferred from the protection seller to the protection buyer based on the recovery rate of the underlying debt instrument.
- Physical settlement: the reference obligation is transferred from the protection buyer to the protection seller, in return for the face value of the underlying debt instrument.

PRICING MODEL

White-Hull Model: Finding the implied probability of default

For the calculation of the default probabilities that were subsequently employed in the forecast of future CDS spreads, we based our results on the White-Hull (2002) CDS pricing model. A default probability of 40% is assumed, as it is widely considered a standard market convention.

The model has been implemented on CDS contracts with a duration of 5 years, using the spreads available on the underlying French, Italian, Greek, and Spanish sovereign bonds. Given CDS are mainly quoted in USD, 1-year US Treasury bills yields were considered as the risk-free rate. The duration of the model spanned from September 20, 2018, to September 17, 2021.

The basis of the model relies on the fact that the CDS buyer and CDS seller are left equally well-off through the transaction. This would translate into the fact that the present value of the expected payments given the credit spread from the CDS buyer (fixed leg) would be equal to the present value of the expected payoff from the CDS seller in case of a credit event (contingent leg).

Fixed leg

It is assumed that if a default occurs, it will take place mid-quarter. Therefore, the estimation of the fixed leg could be expressed through the following summation:

To further break down the process, the following steps were taken:

1. The estimation of the unconditional probability of non-default/survival in year *n*. From Bayesian probability, this can be expressed as the probability of survival in previous quarter $(n-0.25)$ multiplied by the quarterly adjusted default probability in current quarter *n*: $P(S_{n-0.25}) \times P(D_n|S_{n-0.25})$ We assumed that $P(D_n|S_{n-0.25})$, the probability that the entity defaults in year *n* given that the entity did not default in year $n-0.25$, is independent and remains constant, therefore equal to the quarterly adjusted default probability $P(D)$. During the first quarter, the unconditional probability that the entity survives is equal to $P(S_{0.25}) = 1 - P(D)$.
2. To calculate the present value of the quarterly premium payments in year *n*, the discount factor $(\frac{1}{1+risk\ free})^n$ is considered. This is to take into account the time value of money and the duration of the CDS.

1. Therefore, the present value of the quarterly credit spread is proportional to the quarterly probability of survival of the entity multiplied by the discount factor.
2. As mentioned above, it is assumed that, if default happens, it will occur mid-quarter. This means that the CDS buyer will still pay the half-quarter present value of the CDS spread. That would mean that the accrual credit spread paid would be proportional to the quarterly probability of default given survival in the previous period divided by two, times the mid-quarter discount factor.

Therefore, the present value of the expected payments that the CDS buyer is expected to pay (fixed leg) could be expressed the following way:

$$\sum_{i=0.25}^{5} D(t_i) \times q(t_i) \times S \times d + \sum_{i=0.25}^{5} D(t_{i-0.25}) \times q(t_{i-0.125}) \times Df \times S \times \frac{d}{2}$$

$D(t_i) = discount\ factor\ at\ period\ i$
$q(t_i) = survival\ probability\ at\ period\ i$
$S = CDS\ spread\ per\ annum$
$d = accrual\ days\ (0.25\ if\ quarterly\ payments)$
$Df = quarterly\ default\ probability$

Contingent leg

Since we consider that a default will occur mid-quarter, the expected payoff from the CDS seller to the CDS buyer would be equal to the mid-quarterly probability of default given survival in the previous period multiplied by the non-recoverable part of the notional amount. To estimate the present value of that payoff, we multiply by the corresponding discount factor.

Therefore, the present value of the expected payoff from the CDS seller to the CDS buyer could be expressed the following way:

$$\sum_{i=0.25}^{5} (1-R) \times N \times D(t_i) \times Df \times q(t_{1-0.125})$$

$R = recovery\ rate$
$N = notional\ amount$
$D(t_i) = discount\ factor\ at\ period\ \ i$
$q(t_i) = survival\ probability\ at\ period\ i$
$Df = quarterly\ default\ probability$

Estimating probability of default

Since both CDS buyer and CDS seller must equally well-off, it must be that the contingent leg should equal the fixed leg, meaning that the expected CDS payments multiplied by the CDS spread should equate to the expected payoff from the CDS seller

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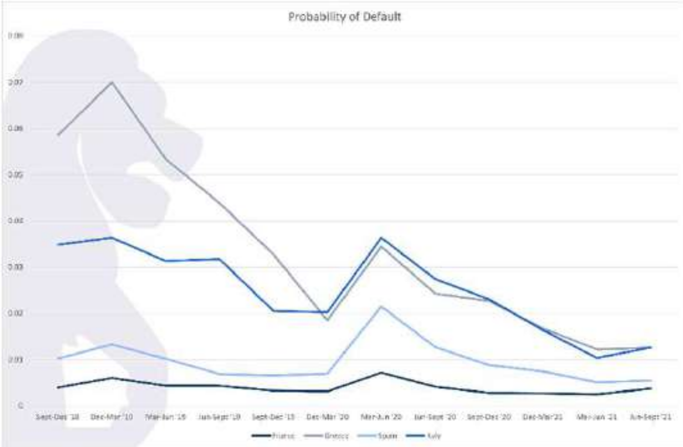
$$\sum_{i=0.25}^{5} D(t_i) \times q(t_i) \times S \times d + \sum_{i=0.25}^{5} D(t_{i-0.125}) \times q(t_{i-0.125}) \times Df \times S \times \frac{d}{2} = (1-R) \times N \times D(t_i) \times Df \times q(t_{i-0.125})$$

Given the CDS spread quote that were taken on quarterly intervals in our specified duration, it is possible to set it equal to S[1] and proceed by solving for the probability of default.
The model has yielded the following results:



The data shows that macroeconomic events have a crucial influence on the default probabilities of sovereign CDS'.
A common trend for default probabilities can be seen, with an increase in the period from December 2018 to March 2019 due to rising tensions between US and China on import tariffs, alongside with an increase in the FED's interest rates during 2018.
However, this increase is immediately reversed, with a general decrease in default probability rates until the Covid-19 outbreak in the first months of 2020. A rapid surge followed by a decline can be registered in all countries between March and June 2020, fueled by uncertainty on Covid effects on economies. The implementation of the Pandemic Emergency Purchase Program from the ECB and similar policies from other central banks were able to reverse the effects of the pandemic on default probabilities. By September-December of 2020 every country's default probability was lower than the initial period of our analysis, and, in the case of France, even below pre-covid levels.
All countries reached their minimum level of default probability during the March-June trimester of 2021, although their default risk started to rise until the last estimation measured for the June-September quarter. The highly anticipated but still unclear tapering measures may be the cause of the increase in risk perceived by the markets.

[1] $S = \frac{(1-R) \times N \times D(t_i) \times Df \times q(t_{i-0.125})}{\sum_{i=0.25}^{5} D(t_i) \times q(t_i) \times d + \sum_{i=0.25}^{5} D(t_{i-0.125} \times q(t_{i-0.125}) \times Df \times \frac{d}{2}}$

CORRELATION WITH OTHER ASSET CLASSES

Due to the definition of the credit contract, credit default swaps show a particular correlation structure with other asset classes on the market. This property may be exploited to diversify the market risk of a portfolio.

A brief reminder of correlation

Correlation is used in statistics to see how two variables move in relation to each other and ranges between the values -1 and 1. The formula is:

$$Cor(X,Y) = \frac{\sigma_{X,Y}}{\sigma_X \times \sigma_y}$$

$$\sigma_{X,Y} = covariance\ between\ variable\ X\ and\ Y$$

$$\sigma_X = standard\ deviation\ of\ variable\ X$$

$$\sigma_Y = standard\ deviation\ of\ variable\ Y$$

In light of portfolio diversification, this concept is useful when attempting to achieve optimisation through the minimisation of volatility. This can be seen through the following formula:

$$Var(aX + bY) = a^2Var(X) + b^2Var(Y) + 2abCov(X,Y)$$

In the case that variables X and Y have a negative covariance, it can be seen their combined variance would be smaller than if positively correlated.

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Portfolio

To estimate the correlation between sovereign fixed income CDS contracts and indexes across different asset classes, a portfolio of equally weighted monthly spreads was created.

With a goal of focusing on the European CDS market, the monthly 5-Year spreads of France, Italy, Greece and Spain were considered for the portfolio, by summing for each month their equally weighted returns.

A second portfolio was built using a Markowitz optimization.

Our Markowitz optimization method takes the classic mean-variance model assumptions:

1. The investor is considered risk-averse. As a rational agent, they try to minimize their risk and maximize their returns.

2. The investor prefers increased consumption, and given they are risk-averse, has a concave utility function.

3. There is perfect information, both regarding the knowledge of the investor regarding market conditions, as well as the ability of the market to absorb changes and updates.

The model states that the investor will aim to pick the portfolio weights that yield the highest return for the lowest possible risk (standard deviation of the portfolio). The expected return of the portfolio with n number of assets can be expressed the following way:

$$E(r_n) = \sum_i^n w_i E(r_i)$$

$E(r_i) = expected\ return\ of\ i_{th}\ asset$

$w_i = weight\ of\ i_{th} asset$

The variance of a portfolio with n number of assets can be expressed the following way:

$$Var(r_n) = \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j Cov(r_i, r_j)$$

For our model, a variance-covariance matrix was created to estimate the covariance between each of the assets returns in the portfolio. As it is evident, the covariance of the returns of an asset with those of the same asset is simply expressed as the variance of that asset.

The Sharpe ratio of a portfolio expresses the extra returns that compensate an investor for the risk they sustain over holding a riskier asset.

The corresponding formula can be constructed the following way:

$$S(r_n) = \frac{E(r_n) - R_f}{Stddev(r_n)}$$

$R_f = Risk\ free\ rate$

$Stddev(r_n) = \sqrt{Var(r_n)}$

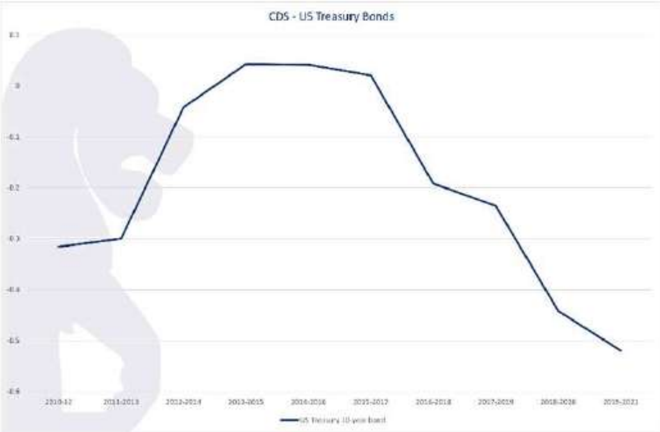
The weights of the portfolio that maximize the Sharpe ratio give us our optimal allocation weights.

In our case, following the application of the optimisation method, a portfolio that contains entirely Spanish 5-Year CDS contracts is created, which can be attributed to the fact that it shares both a medium volatility compared to the riskier Italian and Greek CDS and safer French CDS, but does not completely deviate from the moderately higher returns that riskier sovereign entities are associated with credit default swap markets.

However, this report proceeds considering only an equally weighted portfolio of the 4 credit default swaps. The 3 years rolling correlation between our portfolio's spread and the returns of various asset classes is presented in the next section. The analysis spanned an 11-year period, from January 2011 to November 2021.

Sovereign Fixed Income

The US Treasury 10 Note has a permanent negative correlation with our CDS portfolio during the 11-year period, as can be seen from the following graph:



	US Treasury 10-Year Bond
Portfolio Correlation	-0.282544

Another almost-permanent negative correlation is present between the portfolio and French 5-year bonds, except for the period in between 2013 and 2017 where it was weakly proportional (0.2). This can be related to the aftermath of the European debt crisis and the subsequent aggressive monetary policy of the ECB.

It is interesting to stress out the V-shape line for the relationship with Greece, with the lateral peaks of .6 in 2010-2012 and 2018-2020; the minimum value is instead taken in 2014-2016, with a weak -0.2. The Spanish and Italian 5-Year Bonds appeared to be the only underlying sovereign bonds that showed a consistent positive correlation with the equally weighted CDS portfolio. The Spanish 5-Year bond exhibits a strong correlation of around 0.5 until 2015, with correlation afterwards fluctuating at approximately 0.2. Italian 5-Year bonds have experienced a steep decline since 2020, with the current correlation orbiting around 0.1.

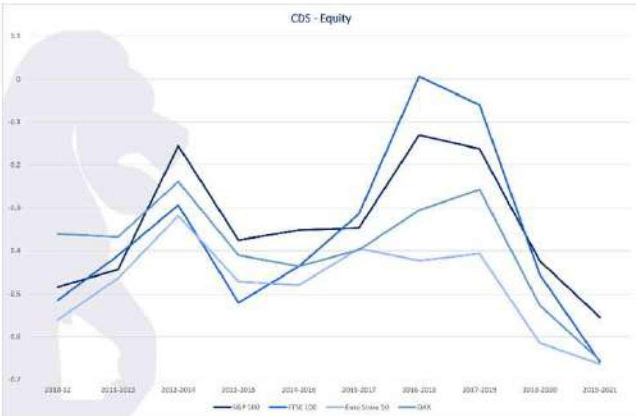
	France 5-Year Bond	Greece 5-Year Bond	Spain 5-Year Bond	Italy 5-Year Bond
Portfolio Correlation	-0.0426	0.0994	0.272449	0.063346

Equity

Our data suggests a permanent negative relationship between the CDS portfolio and the pool of equity indexes considered (FTSE100, DAX, S&P500 and EuroStoxx500). Furthermore, there has been a common trend among all the different equity indexes relationship with CDS, with peaks in 2012-2014 and 2016-2018. Nonetheless it is only in 2019-2021 that the correlation is strongly negative (-0.6).

While the spreads of the four sovereign CDS experienced a momentary peak during the onset of the COVID-19 pandemic in 2020, an immediate slump was exhibited following the announcement of strong fiscal packages and a more coordinated European response. In turn, US equity indexes, following an initial correction, continued to hit new highs, with European indexes following at a more subdued but still positive pace. The course of the stock markets may explain the behaviour of the correlation.

In the 11-year period all the stock indexes show a relatively strong negative relationship with our CDS portfolio.

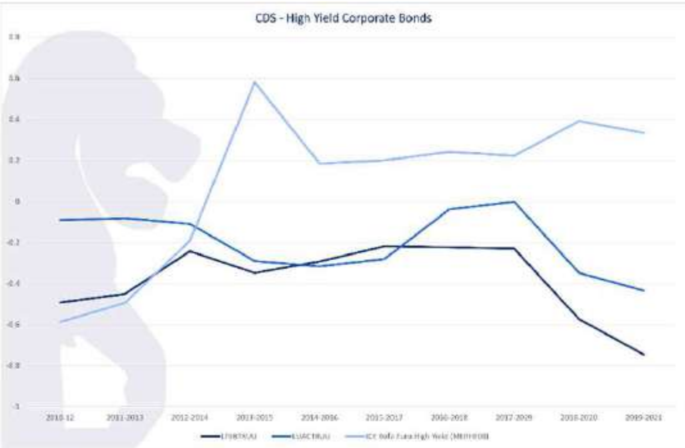


	S&P 500	FTSE 100	Euro Stoxx 50	DAX
Portfolio Correlation	-0.40678	-0.42792	-0.53504	-0.4183

High Yield Corporate Bonds

The correlation between our CDS portfolio and corporate bonds has been mainly negative, besides the "ICE BofA Euro High Yield (MERHE00)" index that in the 2012-2014 period started to show a weak positive relationship with the CDS portfolio (the peak of 0.6 in 2013-2015 is then followed by a maintenance of the positive value).

"LF98TRUU" refers to Bloomberg's US Corporate High Yield Total Return Index. "LUACTRUU" measures the investment-grade US corporate fixed income market. The ICE BofA Euro High Yield Index tracks the performance of Euro denominated below investment grade corporate debt publicly issued in the euro domestic or eurobond markets.

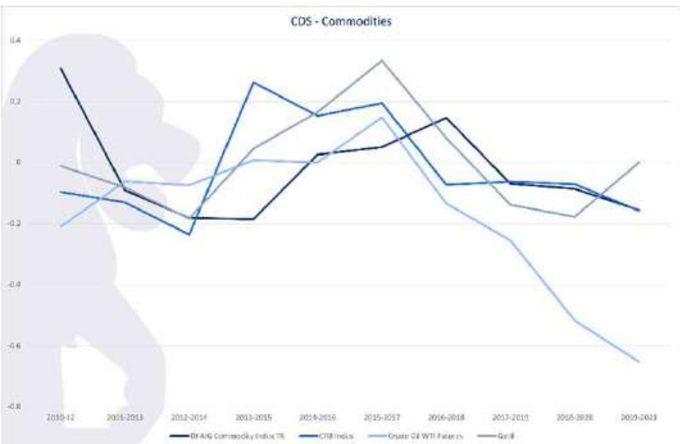


In regard to the general trend of the CDS with both high yield and investment-grade corporate bonds, a weak negative relationship has characterized the period since 2010, the starting date of our analysis.

	LF98TRUU	LUACTRUU	ICE BofA Euro High Yield (MERHE00)
Portfolio Correlation	-0.18784	-0.19163	0.56001

Commodities

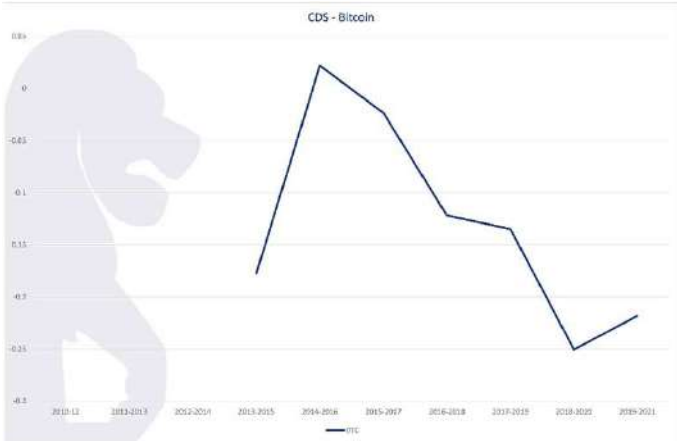
The correlation between our CDS portfolio and commodities fluctuated between -0.2 and 0.4. Only the correlation with oil futures reached a medium negative value in the past recent years. Our data suggests that correlation is weak as it takes both small positive and negative values on a regular basis.



	DJ AIG Commodity Index TR	CRB Index	Crude Oil WTI Futures	Gold
Portfolio Correlation	-0.014428	-0.0344	0.28674	0.04516

Bitcoin

Following the year 2017, correlation between credit default swaps and cryptocurrencies also appears to become increasingly negative. This can be seen through our correlation analysis with Bitcoin, which is evident when considering the exponential growth of its trading price in recent years compared to the gradual stabilisation of CDS spreads following the European debt crisis.



	Bitcoin
Portfolio Correlation	-0.133314

FORECASTING THE FUTURE DISTRIBUTION

The Monte Carlo simulation exploit repeated random sampling to set forth a certain scenario by substituting a range of values (a probability distribution) for any factor that has inherent uncertainty. The analysis that follows has a narrower range than the "what if" analysis. This is because the "what if" analysis gives equal weight to all scenarios, while the Monte Carlo method hardly uses samples taken from the very low probability regions, also called "rare events". Furthermore, Monte Carlo simulation methods do not always require truly random numbers in order to determine a scenario. Many of the most useful techniques use deterministic and pseudo-random sequences in order to make it easy to test and re-run simulation.

For our purposes, we leveraged on this property of the Monte Carlo distribution and used risk free rates and default probability in order to analyse the future possible paths of the spread of the sovereign CDS. To describe the distributions of the simulated variables we decided to model our forecast as a Random Walk with White Noise.

This model does not have particular predictive value, but it easily adapts to data and gives us a treatable description of our forecast's uncertainty.

The model is specified as follows:

$$PD_t = PD_{t-1} + \varepsilon_t \qquad \varepsilon_t \sim N(0,\sigma)$$

$PD_t = Probability\ of\ Default\ at\ time\ t;$
$\varepsilon_t = Error\ at\ time\ t;$

$$RiskFree_t = RiskFree_{t-1} + \pi_t \qquad \pi_t \sim N(0,\theta)$$

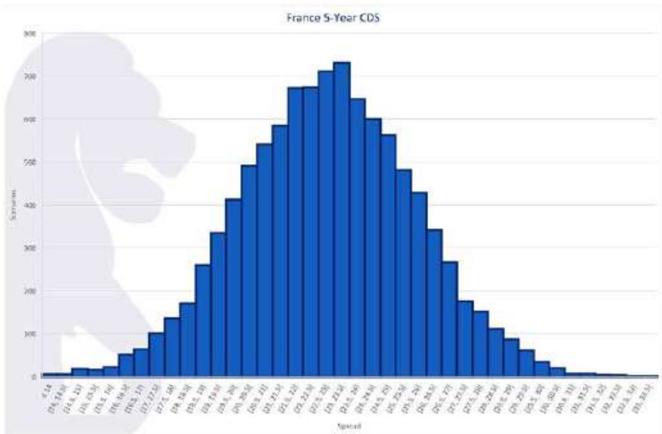
$RiskFree_t = Risk\ Free\ Rate\ at\ time\ t;$
$\pi_t = Error\ at\ time\ t$
A 40% recovery rate market convention is implied in the model.
By taking the quarterly default probability from our pricing model, we computed the change in the default probability rate from September 2018 to June 2021 on a trimestral basis. This period coincides with the last payments of a CDS premium, occurring on the 20th of March, June, September and December. Subsequently, we calculated the standard deviation of all these rates of return.
These operations were repeated for all four countries we are interested in.
As usual, the 1-year US Treasury Bills yield rate was used as a risk free rate, and our assumptions were based on the average yield in the period from 21/06/2021 to 17/09/2021, amounting at 0.073%. As with the probability of default, we proceeded to calculate the average yield from the same trimesters from September 2018 to June 2021.
This allowed us to calculate the rate of return from one period to another and lastly the standard deviation between such rates of return.
Both Default Probabilities and Risk Free Rates don't show a random behaviour on a long time period but on a quarterly horizon a Gaussian Random Walk may be a good approximation of the distribution of minor adjustments.
Our data points take into consideration the troubled period of the Covid pandemic and the subsequent macro policies so the measure of uncertainty of the Default Probabilities we applied are able to capture a possible deterioration of the global health situation and the subsequent increase in default risk for sovereign bonds. On the other hand, the Gaussian Random Walk is unable to take into account the real world limitations of another round of expansionary monetary policy. The naked volatility of our data is really unlikely to be seen in the future since data from the Covid peak is included. We decided to smooth it by computing an adjusted volatility that doesn't take into account the main outliers of the Covid period.

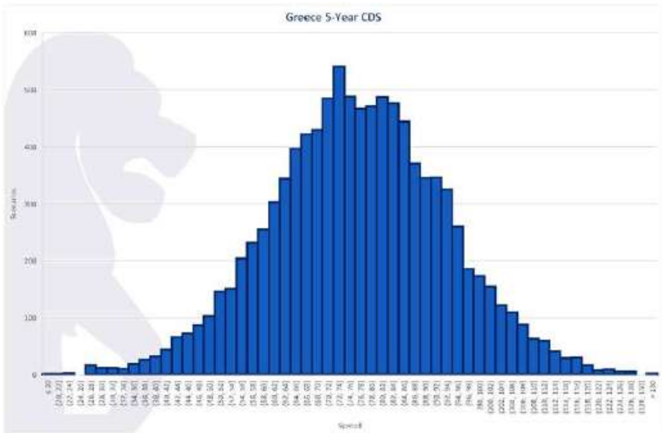
Assuming a 0 mean we computed the sequent volatilities for the errors:

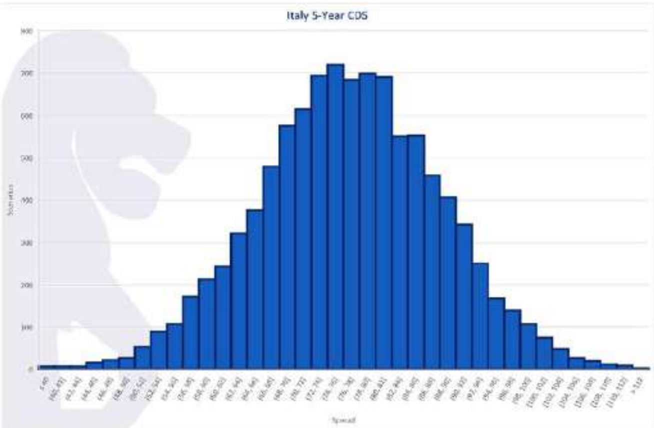
	France 5-Year CDS	Greece 5-Year CDS	Spain 5-Year CDS	Italy 5-Year CDS
Volatility	0.001338	0.018387	0.004391	0.008825

This model presents some issues on its assumptions: our time series is relatively brief and does not make it possible to really test it for the constant variance and absence of autocorrelation in the errors we assume. Another source of possible distortion may be the symmetry of the Normal distribution we used to model the errors of the Default Probability: the extreme values of the left tail of the curve may suggest a negative probability, impossible in the real world. To avoid this inconvenience, a non-negativity constraint was applied to the simulated values. The quantitative distortion induced is minimal.
However, on the other hand such assumptions explain our claim of not using completely random variables for our Monte Carlo distribution, as these inputs were used as a pseudo-random basis for the CDS spread calculation, performed 10,000 times per country, changing according to the abovementioned criteria.

The final results are shown in the graphs below:





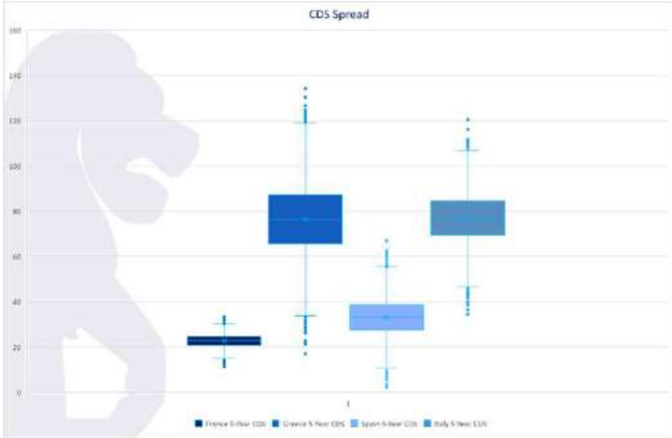


These distributions present a very wide range of possible values that the CDS price can take in the upcoming periods. The estimates we computed may be useful to evaluate the risk of adding CDS to a portfolio.

	France 5-Year CDS	Greece 5-Year CDS	Spain 5-Year CDS	Italy 5-Year CDS
Volatility	2.7978 bp	16.1873 bp	8.3974 bp	11.2940 bp

	France 5-Year CDS	Greece 5-Year CDS	Spain 5-Year CDS	Italy 5-Year CDS
Minimum	11.36300	17.07185	2.371555	34.45425
Maximum	33.25452	134.25920	67.00292	120.4585

The countries with historical economic fragility show a greater volatility in their spread since markets expect their default probabilities to be more sensible to macroeconomic news, with greater price adjustments as a result.
To help us understand better the possible scenarios and have a general picture of 4 countries altogether, it is worth having a boxplot view of the phenomenon.



Even though the Italian and Greek CDS have almost the same mean, we can note that the Italian CDS presents a larger fluctuation, which makes them the most volatile across the four countries. Italian CDS are followed in volatility by Greek and Spanish CDS, which, despite having a lower price, approach the same width of range as the Greek CDS. Lastly, it is worth noticing how French CDS have a considerably lower volatility compared to the other countries, in addition to a lower spread.

This observation therefore gives space to possible outliers: values which are either lower than the first quartile less than 1.5 times the interquartile range or higher than the third quartile added to 1.5 times the interquartile range. The total number encountered during the 10.000 simulations are reported below:

	France 5-Year CDS	Greece 5-Year CDS	Spain 5-Year CDS	Italy 5-Year CDS
Lower Outliers	30	54	39	44
Upper Outliers	24	35	39	31

The presence of outliers is between 0.5% and 1% of all the possible values a CDS spread can take.

CONCLUSION

The analysis of the market behaviour of the CDS spread showed some interesting results on the correlation with different asset classes. The correlation between equity and our CDS portfolio is mainly negative. The results with high yield bonds indices are similar while the analysis of the interaction with commodities does not show a clear direction, but small levels of correlation were found.

Our pricing model was able to provide historic data on the probability of default implied in CDS market prices. Using the historic volatility, a forecast of the future distribution of spreads was built. A simple Random Walk with White Noise Model was implemented to model the path of the CDS spreads. The results, together with the correlation analysis, provide some insight on the effect on risk and return of adding such instruments to a wider portfolio.