MIMS – Disentangling Cryptocurrencies

Research Team

Report for Hercle Financial – December 2021

Head of Research

Tommaso Beverina

Research Analysts

Cisem Almali

Deniz Aktas

Huiying Song

Jacopo Prevedello

Thomas Ferradini

Over the last three years, cryptocurrencies have become one of the best performing asset classes in the financial markets and have currently reached an outstanding market capitalization of more than $ 2tn.

In this report, produced in collaboration with Hercle Financial, a team made up of members from different divisions of Minerva analyses the correlation between the main cryptocurrencies by market cap and several other asset classes, such as commodities, traditional currencies, government bonds, and other cryptos. This aim is achieved by applying different correlation models, ranging from simple correlation statistics to linear and stationary dynamic linear models.

Hercle developed proprietary ultra-low latency trading technology to execute High-Frequency market making strategies and to offer an end-to-end prime-brokerage solution to institutional investors who are interested in gaining exposure to crypto-assets.

SUMMARY

Introduction

Cryptocurrencies' Market Structure

The cryptocurrency market is relatively young, with Bitcoin being its "founder". This asset class is extremely specific and unique, which means that standard market metrics cannot be applied to it. This is why unique indicators are being actively developed for market analysis to assess the cryptocurrency market.

One of them is the BITCOIN dominance index. For a long time, until 2017, Bitcoin had the status of absolute dominance of the market with a share of over 80% in terms of capitalization. The dominance index measures the share of Bitcoin in the total capacity of the cryptocurrency market and the calculation is based on data for all digital currencies, including newly formed ones. For an objective assessment of activity in the cryptocurrency market, one should study the dynamics of the daily turnover of this sector. As a rule of thumb, an increase in market turnover indicates an increase in the liquidity of cryptocurrencies. Therefore, it should have a positive effect on their value as means of payment.

Bitcoin and Ethereum hold the leading positions in the trading volume with shares of 26.27% and 14.06%. The main features of these currencies are their longer history in the market, high capitalization, and high volatility. However, the market is constantly seeing the introduction of new cryptocurrency systems and the constantly growing competition actively contributes to its development. As a result, the Bitcoin dominance index has been steadily deteriorating over the years, reflecting the weakening of its leadership position. There is strong competition in the industry, which constantly stimulates and improves its growth. To assess the state of the market, it is necessary to consider the dynamics of market capitalization, as well as liquidity and volumes of trading.

Market Capitalization

For a cryptocurrency like Bitcoin, market capitalization is the total value of all the coins that have been mined. It's calculated by multiplying the number of coins in circulation by the current market price of a single coin at any given time. One way to think about the market cap is as a rough measure of how stable an asset is likely to be. Ceteris paribus, larger market cap indicates that the investment is more stable. However, it is important to note that many cryptocurrencies' market cap can swing dramatically due to their volatility.

Cryptocurrencies are classified by their market cap into three categories:

- *Large-cap cryptocurrencies*: market cap higher than $10 billion, as they have demonstrated a track record of growth and often have higher liquidity - meaning they can withstand more people cashing out without the price being dramatically impacted - investors consider them to be lower-risk investments. As of November 2021, 12 cryptocurrencies are included in this category.
- *Mid-cap cryptocurrencies*: market cap between $1 billion and $10 billion, generally considered to have more untapped potential but also higher risk.
- *Small-cap cryptocurrencies*: market cap of less than $1 billion, the most susceptible to dramatic swings based on market sentiment.

Sidenote: it might occur to see references to the "circulating supply" market cap, the one considered above, or to the "fully diluted supply" market cap. The former indicates the amount, which is currently in circulation, while the latter indicates the amount that will eventually be mined. To put it into perspective, Bitcoin has 18.8 million coins in circulation as of November 2021, while the "fully diluted supply" is equal to 21 million.

The five largest cryptocurrencies by market cap

The five largest cryptocurrencies by market cap are Bitcoin, Ethereum, Binance Coin, Tether and Solana.
Bitcoin's unique feature is the fact that it was the very first cryptocurrency to appear on the market, managing to create a global community and give birth to an entire new industry. It has established a conceptual and technological basis that inspired the development of thousands of completing projects.

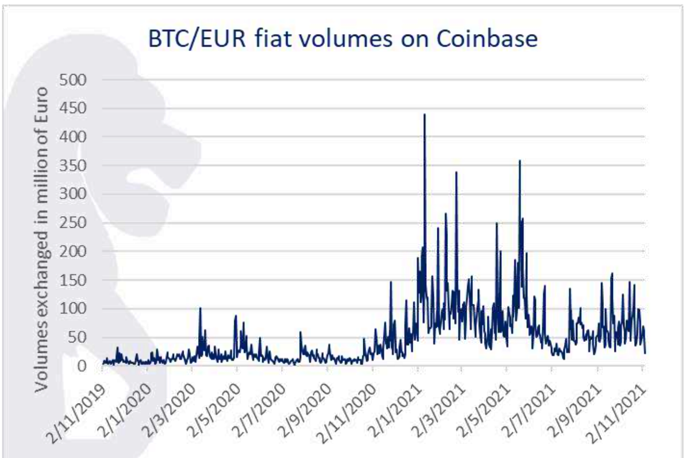
On the other hand, Ethereum has pioneered the concept of a blockchain smart contract platform; smart contracts are computer programs that automatically execute the actions necessary to fulfill an agreement between several parties on the internet: they were designed to reduce the need of trusted intermediates between contractors, reducing transactions costs and increasing transaction reliability. Ethereum has precisely designed a platform that allows to execute such contracts using the blockchain.
Binance Coin is a cryptocurrency launched by the biggest cryptocurrency exchange globally, Binance: it went through a significant price increase at the beginning of 2021, which has put it on the map of investors in this sector.
Tether is an example of stablecoin: its value is pegged to the US dollar. Whenever new tokens are issued, the same amount of USD is allocated to its reserves, ensuring that the cryptocurrency is fully backed by cash or cash equivalents.
Finally, Solana is notable for the incredibly short processing times the blockchain offers. Its hybrid protocol allows for significantly decreased validation times for both transactions and executions of smart contracts. This unique feature has attracted a lot of interest from institutional investors.

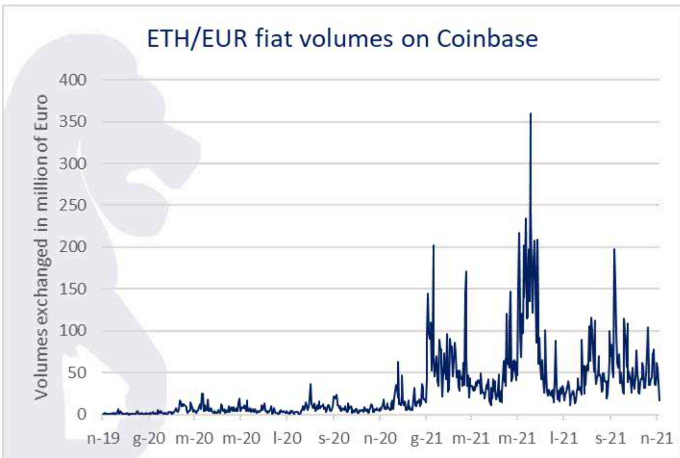
Volume

Volume can serve as a prediction of future price and its demand (e.g., an increase in trading volume is generally considered a precursor to a big price move). It is an extremely important indicator for traders to determine future price patterns.

Volume is associated to the concept of liquidity: the former is the sum of actual trades taking place, while the latter is the amount available for trading at any single price. Usually, the higher the volume of cryptocurrency transactions, the more liquid the crypto market will be. Greater volumes of cryptocurrency transactions reduce the chance of distorted pricing and generally lead to a fairer value. On the contrary, a low volume of exchange signals inefficient pricing as it is more probable that the asking price of sellers fails to meet the bids of potential buyers. Moreover, high trading volumes help avoid drastic price movements in price after a significant sale, and they are considered as testament to the trustworthiness of a cryptocurrency.

As it is difficult to find aggregate data on volumes in the cryptocurrency market, we analyzed the time series of the volumes BTC/EUR and ETH/EUR exchanged on the platform Coinbase with daily frequency in the last two years. It is interesting to note how the two distributions seem to be tightly correlated, as we can clearly see from the major spikes.





Are Cryptos' high trading volumes a scam?

While low volumes exchanged create great arbitrage opportunities for investors, the main beneficiaries of high trading volumes are the cryptocurrency exchanges that are making profits with trading fees on transactions. As the market is still under-regulated, a problem in the measure of volumes has emerged: some cryptocurrency exchanges have been faking their volume numbers in order to raise the visibility of their businesses and bring in more customers.

This practice is called wash trading, a process whereby a trader buys and sells a security for the express purpose of feeding misleading information to the market. In some situations, wash trades are executed by a trader and a broker who are colluding with each other, and other times wash trades are executed by investors acting as both the buyer and the seller of the security.

In 2018 the trader Sylvain Ribes, after extensive research, concluded that approximately 93% of OKEx's volume, a China-based exchange that had among the highest trading volumes, was fabricated. Experiments at other cryptocurrency exchanges revealed similar data points. At Huobi, another big China-based exchange, he estimated that 81.2% of trading volume was fake. HitBTC and Binance, which is arguably the biggest crypto trading platform, showed a similarly large slippage amount.

According to Sylvain Ribes a bit of wash trading and artificial volume inflation is to be expected in a thoroughly unregulated market, but the magnitude of these findings forces us to reconsider the validity of trading volumes as a metric in the cryptocurrency market. Although some financial media and websites that cover cryptocurrencies have started a campaign to force exchanges to report real numbers, no concrete policy has been implemented to solve this issue yet.

Liquidity

A liquid market is one with many available buyers and sellers and comparatively low transaction costs. The details of what makes a market liquid may vary depending on the asset being exchanged. In a liquid market, it is easy to execute a trade quickly and at a desirable price because there are numerous counterparties and the product being exchanged is standardized and in high demand. In a liquid market, despite daily changes in supply and demand, the spread between what the buyer wants to pay and what sellers will offer remains relatively small.

While liquid markets are deeper and smoother, an illiquid market can put traders in positions that are difficult to exit. The liquidity problem is one of many factors that lead to sudden movements in the Bitcoin price, and improved liquidity could help to reduce its risk and that of other cryptocurrencies. The opposite of a liquid market is called a "thin market" or an “illiquid market". Thin markets may have considerably large spreads between the highest available buyer and the lowest available seller.

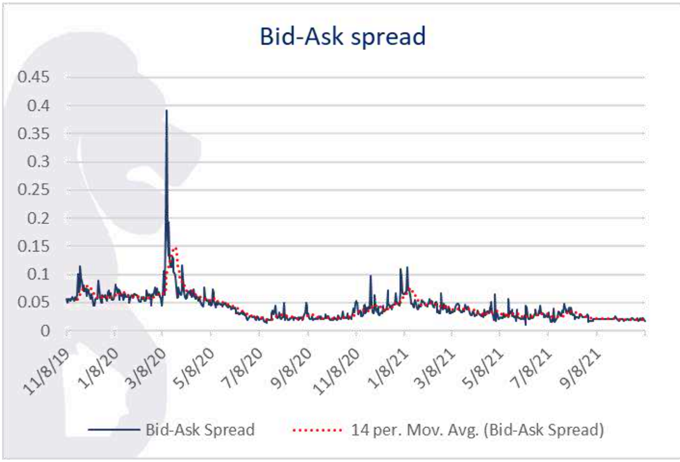
One significant factor related to liquidity is volatility. Low liquidity can generate high volatility when supply or demand changes rapidly; conversely, sustained high volatility could drive some investors away from a particular market. Whether it be correlation or causation, a market that has less liquidity is likely to become more volatile.

One way of defining liquidity is the ability of an asset to be converted to cash on demand. Another view is that liquidity is determined by the bid-ask spread, and an investment with a lower bid-ask spread has higher liquidity. The bid-ask spread is an important metric when assessing an exchange in that it represents the costs of immediately buying or selling a security. Bid-ask spreads are usually calculated using high-frequency intraday data that are both expensive to purchase and time-consuming to process.

While trading has become relatively frequent in cryptocurrencies the liquidity of these markets is difficult to determine. The lack of a consolidated feed coupled with the high number of exchanges and jurisdictions makes it difficult to calculate high-frequency bid- ask spreads thereby hampering the comparison of liquidity across cryptocurrency exchanges.

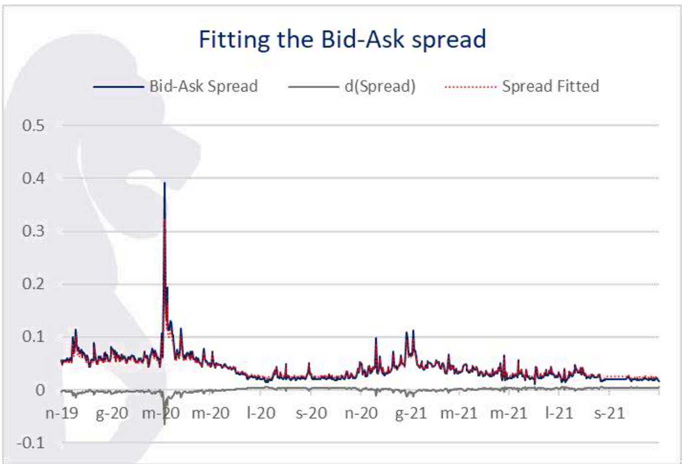
Liquidity of Bitcoin

To assess the liquidity of Bitcoin, we analysed the time series of the bid-ask spread over BTC/EUR with daily frequency in the last two years.



The moving average highlighted in red shows the clear downward sloping trend throughout the period. Moreover, the two major spikes of March 2020 and January 2021 respectively correspond to the outbreak of the COVID-19 pandemic and to the concerns for the rising inflation.

An important measure for the bid-ask spread is the "resilience", which represents how quickly the spread revert to the long-term average after a spike.



We implemented a simple mean reverting model based on a stochastic process. In particular, we used the Vasicek stochastic equation for the short-term rate with no variance component, according to the following equation:

$$d(Spread) = \alpha * (long\ term\ average - current\ spread)$$

Running the regression model, we minimize the SSE (Sum of squared estimate of errors) reaching a value of α=0.042.To interpret the result, note that α=1 means that the model instantaneously reverts back to the long-term average, while α=0 means that the model is not mean reverting. Thus, the BTC/EUR spread does not seem to have a high resilience. However, this result is clearly strongly impacted by the huge spike of March 2020 due to the Covid outbreak.

Crypto exchanges

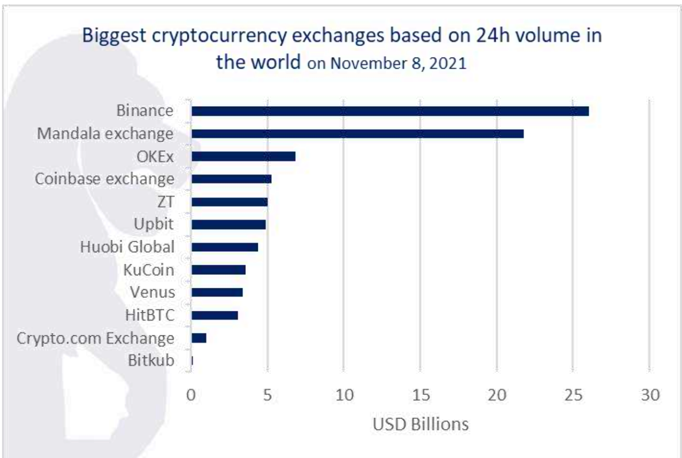
Despite the libertarian promises of fully decentralized transactions and aspiration towards a democratized access to financial markets, crypto assets still heavily rely on intermediation for important aspects of secondary market trade execution and settlement.

Indeed, many of the largest crypto exchanges are for-profit businesses, collecting fees to facilitate crypto assets trading and then distributing profits to the individuals who own the platforms. Such exchanges are proprietary, permissioned blockchain ledgers that execute transactions using efficient operational procedures: users deposit their funds in a pooled wallet directly controlled by the exchange, which then engages in matching buy and sell orders. The centralized exchanges create accounts that store user funds and generally enable traders to execute, clear, and settle buy/sell orders.

The largest crypto exchanges in the world operate in this manner. As it is possible to note from the graph, the largest by far in terms of daily trading volumes is Binance with $26.03 bn, founded in 2017 by Changpeng Zhao. Since it was banned in the US in 2019, it has been created Binance.us to comply with US laws.In second place, with $21.76 bn, is Mandala Exchange, the first to be launched on the Binance Cloud platform.

Following, among the most notable are Coinbase and Crypto.com, with respectively $5.24 bn and $1.02 bn of daily trading volumes. The former made the headlines earlier this year when it became the only publicly listed cryptocurrency exchange at a staggering valuation of $76 bn, close to that of BNP Paribas. The latter has instead seen its revenue grow 20-times this year and has recently paid $700 m to acquire the Staples Center's naming rights, which is now called Crypto.com Arena.

Thanks to soaring cryptocurrencies prices, companies like Coinbase and Crypto.com have turned into billion-dollar enterprises with high margins, surfing on an influx of new investors. Unlike traditional markets, crypto exchanges can charge customers 0.4% on transactions that take place on the venue and even more if trading takes place on the company's mobile app. Moreover, they typically charge investors less as the size of trades increase, incentivizing retail traders to take bigger risks by putting more money into their accounts and using leverage.



Source: https://www.statista.com/statistics/864738/leading-cryptocurrency-exchanges-traders/

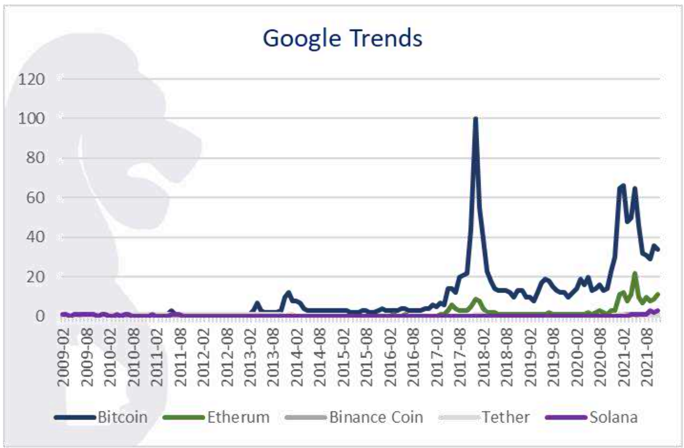
Cryptocurrencies and Google searches

We used Google Trends to get some insights on the public interest towards the five largest cryptocurrencies by market capitalization. For those who are not familiar with it, it is a Google service that allows to identify the general trend of a specific search term and analyze correlations.

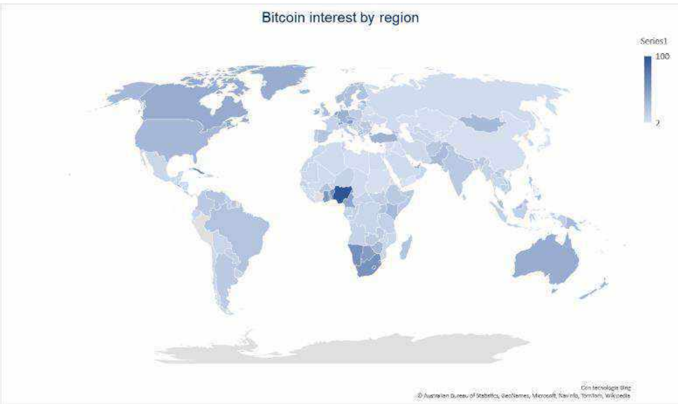
From the first graph it is possible to note that since the mining of the first Bitcoin block (January 3, 2009), Bitcoin has largely outperformed the other digital currencies in terms of Google searches: the first crypto ever created is still the most important driver of the whole crypto-asset industry. Moreover, over the last year, Ethereum has greatly increased its relevance among Google users, following a strong growth in its price that started in October 2020.

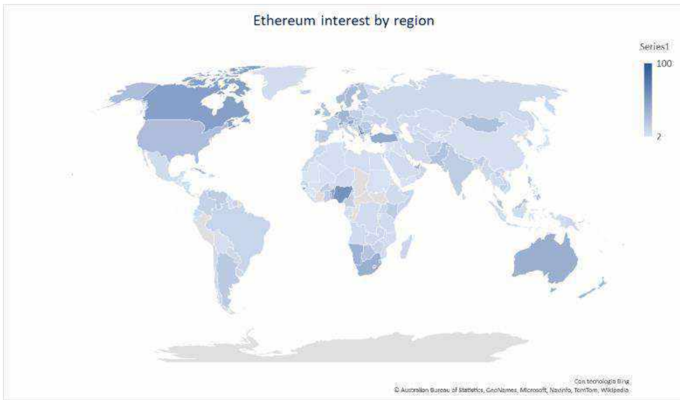
On the other hand, the interest towards Binance Coin, Tether and Solana has been relatively insignificant, compared to Bitcoin and Ethereum. This shows how trends tend to focus more on the most well-known cryptos rather than on "niche" digital currencies.

Further analyzing the graph, it is possible to see a steep spike of Bitcoin-related searches in the period ranging from April to December of 2017, during which its price increased by an astonishing 1600% and its popularity exploded. Ethereum experienced a tremendous growth in recognition over the second half of 2017, following a rally that abruptly took the price of the cryptocurrency from $18 in March 2017 to $1400 in January 2018.



One more peculiar data to highlight is the interest by region for the largest cryptocurrencies: surprisingly, in first place for Bitcoin is Nigeria, with Lagos leading the ranking for cities. Following are Swaziland, Netherlands Antilles, Curaçao and Cuba. On other hand, Kosovo is leading the leaderboard for Google searches of Ethereum, followed by North Macedonia, Liechtenstein, Gibraltar, and Singapore.



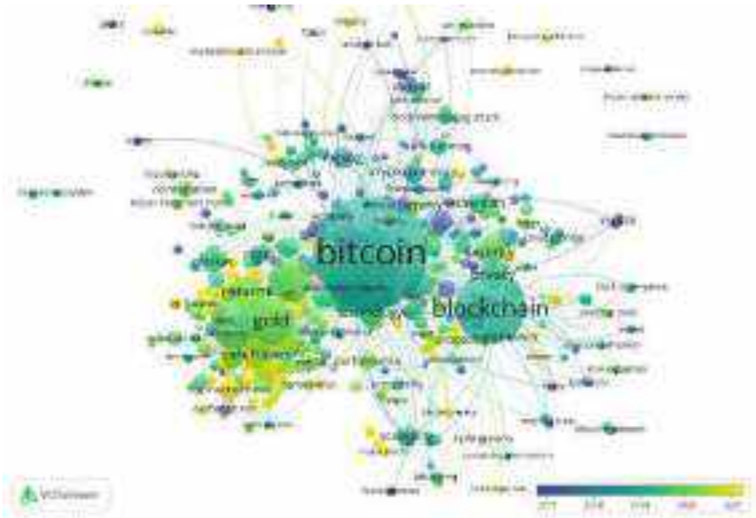


This highlights a crucial and recent trend: crypto is quietly building roots in the developing countries, especially in the ones that have a history of financial instability or where the barriers to accessing traditional financial products such as bank accounts are high. Cryptocurrencies find fertile ground in such countries often because national currencies cannot serve as an effective store of value, means of exchange or unit of account due to unpredictable inflation and fast-moving exchange rates, clunky and expensive banking systems, financial restrictions, regulatory uncertainty and, above all, existence, or threat of financial controls.

Cryptocurrencies and research papers

To compare the interest of the broader public with the academic perspective, we used Vos Viewer, a software developed by Leiden University for constructing and visualizing bibliometric networks. It allows to analyze the popularity achieved by cryptocurrencies in research papers, in a manner like the one provided by Google trend for the online searches.

As it is possible to note from the map below, Bitcoin is undoubtedly the most connected word in academic papers related to cryptocurrencies: it is once again a strong acknowledgement of the tremendous importance of the first crypto ever created. Moreover, the map gives another important piece of information: the average date of publication of papers containing the word "bitcoin" is late 2018. Thus, academic circles have been forced to confront with the increasing relative weight of the cryptocurrency approximately since its massive surge in price over the second half of 2017. Furthermore, the majority of the terms contained in the map is part of articles that have been published during the last two years, symptom of the recent overwhelming rise in popularity of the crypto realm which had to be acknowledged by the academic world.



CORRELATION ANALYSIS OF BITCOIN

Correlation Theory

Pearson Correlation (Parametric)

The Pearson correlation method is used to assess the linear relationship between the pairs of continuous variables. The outcome of the test is denoted by “r” which is an indicator of both the strength and the direction of the relationship between the variables. The aim of the Pearson correlation test is to evaluate whether a linear relationship exists between two set of data, and its degree if the existence holds, by drawing a line of best fit through the data of the variables that are being tested.
The Pearson correlation coefficient ranges from -1 to 1. While the sign of the coefficient indicates the direction, the magnitude is a measure of the strength of the relationship. The test results in a positive coefficient if the value of one variable increases as the other variable increases, and in a negative coefficient if the other variable is inclined to decrease. As r gets closer to 1 in absolute value, a stronger relationship is suggested. The coefficient also can take a value of 0 that implies no linear association between the two data sets. The underlying assumptions for the Pearson correlation are:

- Both variables should be normally distributed
- There should be no significant outliers
- Continuity
- Linearity
- Homoscedasticity

The distribution of the two variables is approximately normal if the distribution follows a bell-shaped curve pattern. Outliers are the single observations that fail to track the dominant pattern of the dataset. Linearity is examined to determine whether the observations fall on a “straight line” in the scatter plot of the variables. Homoscedasticity holds in case of the data is equally distributed around the regression line.
The Pearson correlation is calculated by the formula presented below.

$$r_{xy} = \frac{\sigma_{xy}}{\sigma_x * \sigma_y}$$

Kendall Correlation (Non-Parametric)

Kendall correlation is a rank-based correlation method used as an alternative to Pearson correlation when at least one of the assumptions for the Pearson correlation fails to hold. It is also the best alternative to Spearman correlation in the cases the number of observations is small, and the data has many tied ranks, i.e., some of the observations in a data set have the identical value that creates the issue of determining their ranks.
Kendall correlation is a measure of ordinal association between two variables in which the order of the observations is taken into account in terms of their quantities. The Kendall correlation coefficient is reported as “tau (τ)”, can range from -1 to 1, and it increases as the similarity of the ranking improves between the two data set. The coefficient is calculated based on the concordant and discordant pairs. If the difference between the two observations taken from a dataset has the same sign as the difference between the two observations taken from the other dataset, the pair of the observations is considered to be concordant. Similarly, the pair is discordant when the sign changes. The following assumptions should be checked before the Kendall’s rank correlation is applied:

- Ordinality or continuity
- Monotonicity (desired)

Non-numeric can be ordered inherently if the ordinality holds such as grouping income level into low/medium/high. For monotonicity the relationship between the two variables should be consistent in general, as one of the variables increase the other one should too, and vice versa.
The formula for the Kendall's tau coefficient is the following:

$$\tau = \frac{n_c - n_d}{\frac{1}{2} \times n \times (n-1)}$$

n_c *denotes the number of concordant pairs*
n_d *denote the number of discordant pairs*

Spearman Correlation (Non-Parametric)

Spearman correlation is a rank-based correlation method, i.e., it assesses the relationship between two variables on ordinal scale. The method might be used when the assumptions for the Pearson correlation are not met. The Spearman correlation coefficient is denoted by "rho (ρ)" which measures the direction and strength of the relationship between the ranked variables. The coefficient may take values between -1 and 1. The interpretation of the coefficient is identical to the Pearson correlation case. The assumptions for the Spearman correlation are the same as the Kendall's correlation assumptions. The coefficient is calculated by the following formula:

$$\rho = 1 - \frac{\delta \sum d_i^2}{n(n^2-1)}$$

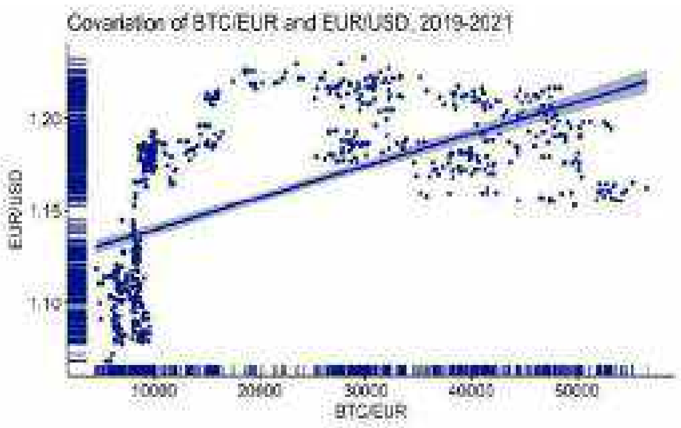
d_i*=the difference between the ranks of corresponding variables*
n*= number of observations*

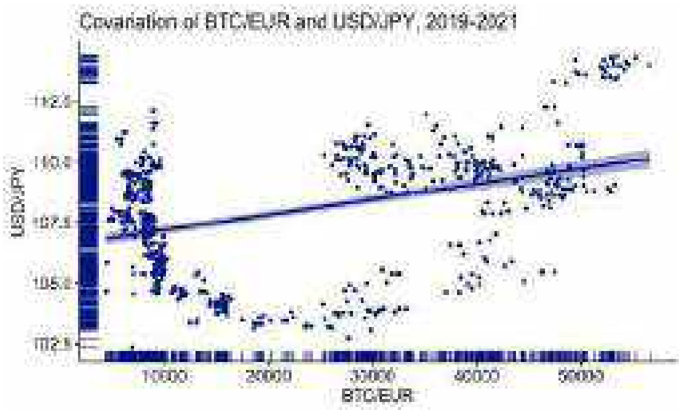
BTC/EUR

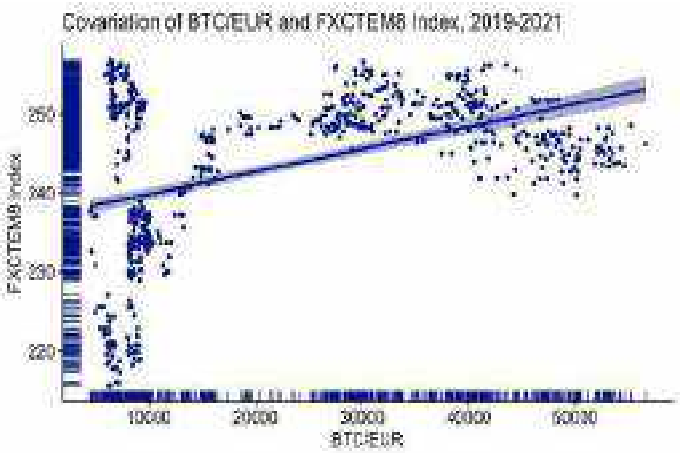
Pearson's product-moment correlation, Kendall's rank correlation and Spearman's rank correlation tests will be conducted. Before applying the three correlation tests (Pearson, Kendall, Spearman), the assumptions underlying the tests will be checked.

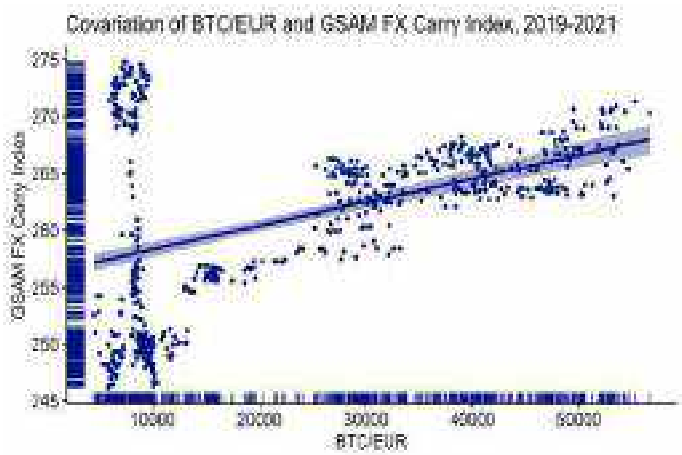
FX

Linearity Tests







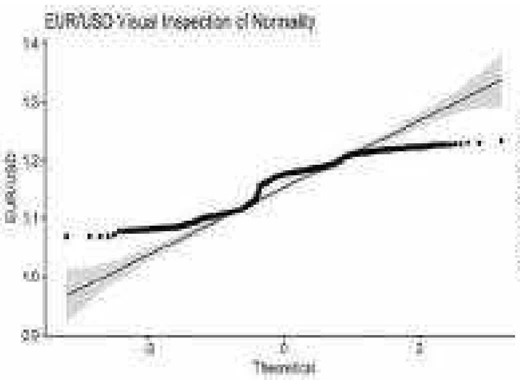


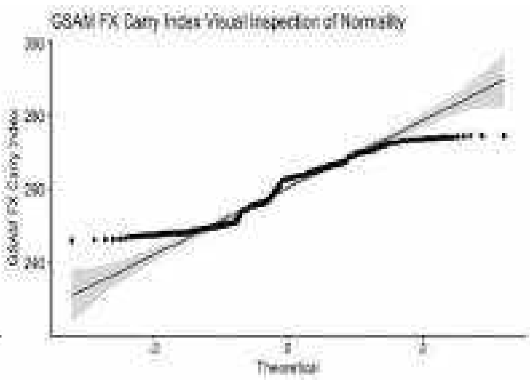
The covariation graphs of BTC/EUR and EUR/USD, USD/JPY, FXCTEM8 Index, GSAM FX Carry are plotted to visualize the nature of the relationships. As clearly can be seen in the graphs above, the relationship between the prices of BTC/EUR and the variables lacked linearity in the analysis horizon. On top of not following a fitted line, the observations also fall around the line in an irregular pattern.

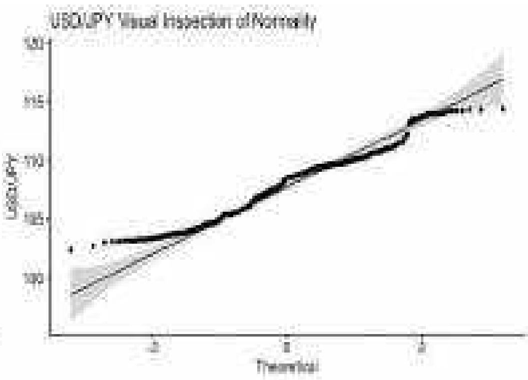
Normality tests

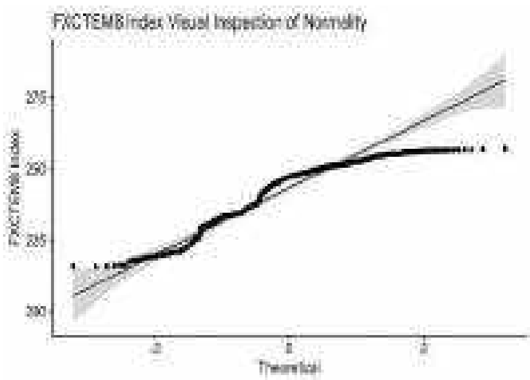
Test	Variable	Statistic	P-value
Shapiro-Wilk normality	BTC/EUR	W = 0.83207	p-value < 2.2e-16
Shapiro-Wilk normality	EUR/USD	W = 0.91668	p-value < 2.2e-16
Shapiro-Wilk normality	USD/JPY	W = 0.97119	p-value = 7.515e-11
Shapiro-Wilk normality	FXCTEM8 Index	W = 0.90242	p-value < 2.2e-16
Shapiro-Wilk normality	GSAM FX Carry	W = 0.94148	p-value < 2.2e-16

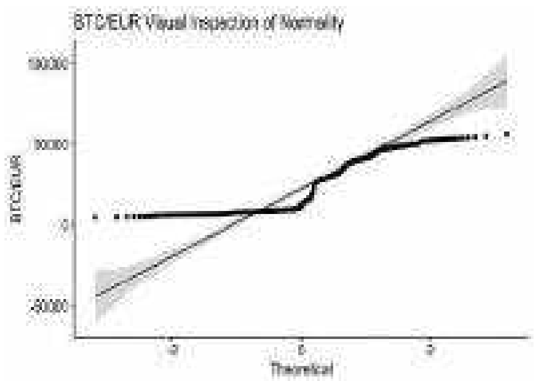
Shapiro-Wilk normality test is applied to all the five datasets to ascertain whether they are normally distributed or not. In all the cases, the test concludes in a p-value much smaller than 0.01. Since the values do not fall in the 99% confidence level, the null hypothesis -the data is normaly distributed, is rejected. Hence, it is safe to state that the prices of BTC/EUR and EUR/USD, USD/JPY, FXCTEM8 Index, GSAM FX Carry are not normally distributed in the analysis period. The conclusion is further supported by the following visual tests. In the graphs the theoretical normality line and the actual observations are compared. Although USD/JPY appears to be approximately normally distributed in the visual inspection, the p-value of 7.515e-11 pertaining to the Shapiro-Wilk normality test is small enough to reject the possibility. Thus, returns are modelled.







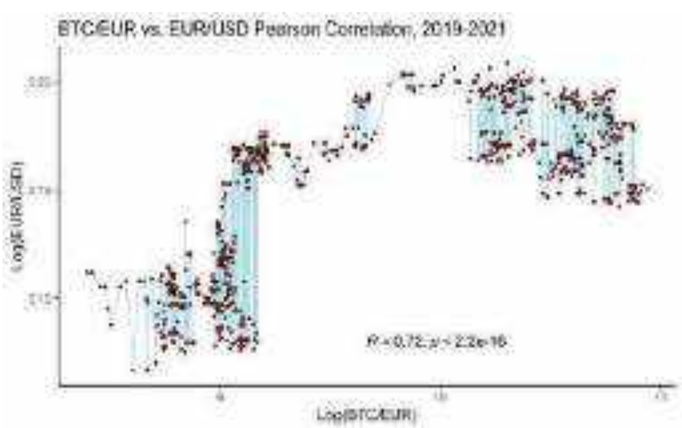




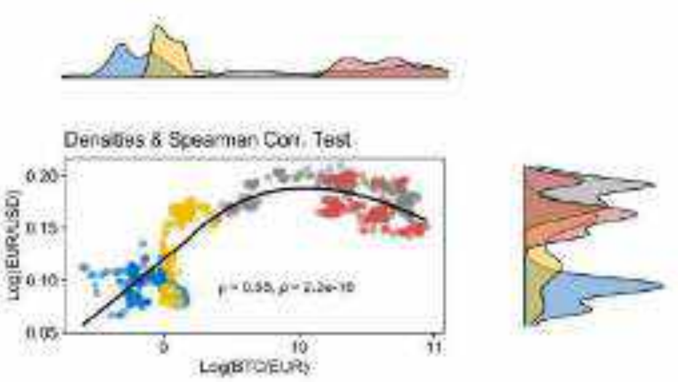
Correlation with the EUR/USD

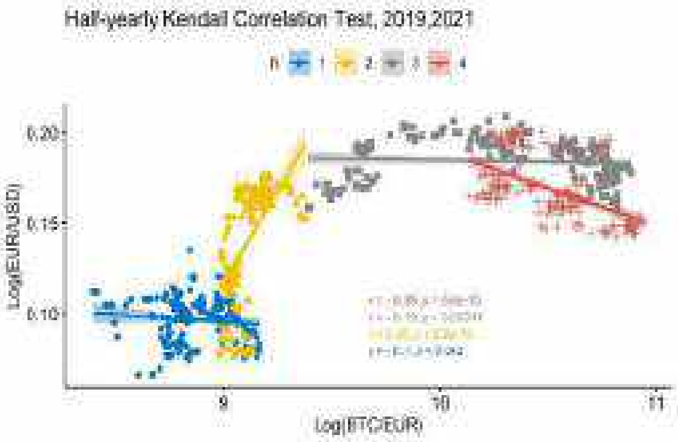
Test	Variable	Statistic	P-value	Sample Estimate
Pearson's product-moment correlation	BTC/EUR ~ EUR/USD	t = 28.1	< 2.2e-16	0.720
Kendall's rank correlation tau	BTC/EUR ~ EUR/USD	z = 18.733	< 2.2e-16	0.462
Spearman's rank correlation rho	BTC/EUR ~ EUR/USD	S = 20854894	< 2.2e-16	0.684

Considering the sample estimates of coefficients, the Pearson's product-moment correlation has the highest value with 0,72. The p-value of the test is smaller than 2.2e-16, that bolsters the rejection of the null hypothesis which claims the coefficient is equal to 0, hence no linear relationship between the pair. Nevertheless, in BTC/EUR vs. EUR/USD case, several of the assumptions of the Pearson test do not hold such as normal distribution and linearity. Due to non-normal distributions, logarithmic transformation is applied to the data before the correlation tests.



For the other tests, the assumptions are much loose: continuity and monotonicity (desired). Continuity holds for both variables. Monotonicity is partially met that can be observed from the plot on the left. Since it is not required absolutely, the use of Kendall and Spearman tests would yield in more meaningful interpretations.



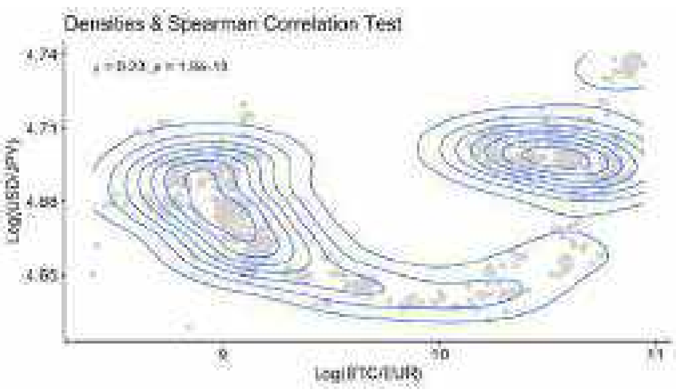


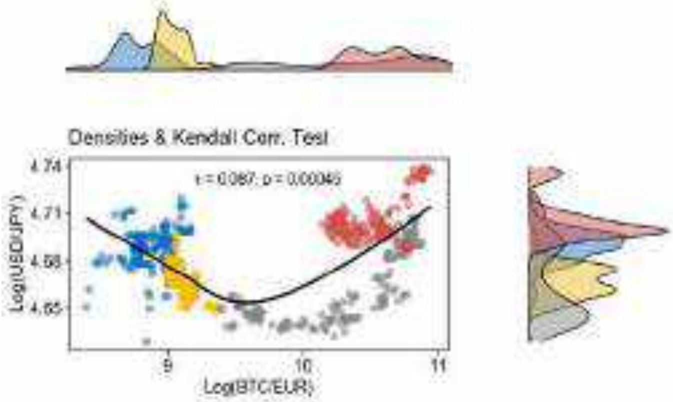
Both the coefficient rho of the Spearman and tau of Kendall tests are positive, where the tests are statistically significant even with the 99% confidence level. BTC/EUR and EUR/USD are significantly correlated when tested ordinally. Rho is approximately 0.22 higher than tau. The reason for the difference may be explained by the tied ranks that decreases the efficiency of the rank-based tests. The tests conclude in that while the crypto currencies are considered risky compared to conventional currencies, BTC/EUR are positively related in the last two years.

The half-yearly calculated Kendall tau's for BTC/EUR vs. EUR/USD can be seen on the plot on the right side. Except the second half year of 2020, all the half years has negative correlation coefficients. Yet, the tau coefficient for the entire horizon (0.46) is almost the same as the second half year's coefficient (0.45). Therefore, the rankings in the second half year reflect most of the weight in the Kendall's tau calculations.

Correlation with the USD/JPY

Test	Variable	Statistic	P-value	Sample Estimate
Pearson's product-moment correlation	BTC/EUR ~ USD/JPY	t = 8.1122	2.091e-15	0.287
Kendall's rank correlation tau	BTC/EUR ~ USD/JPY	z = 3.509	0.0004497	0.086
Spearman's rank correlation rho	BTC/EUR ~ USD/JPY	S = 50801057	1.818e-10	0.232



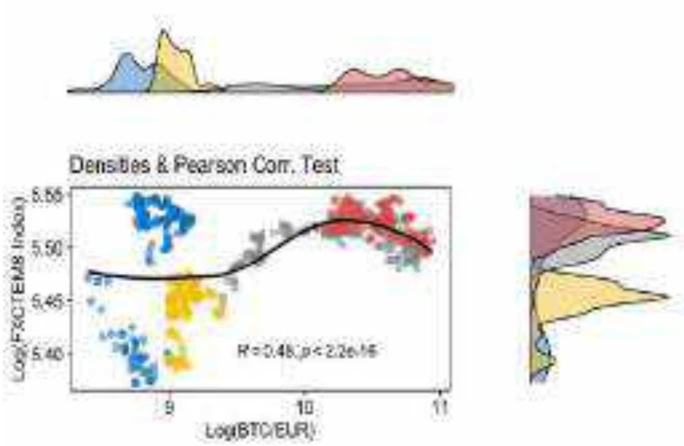


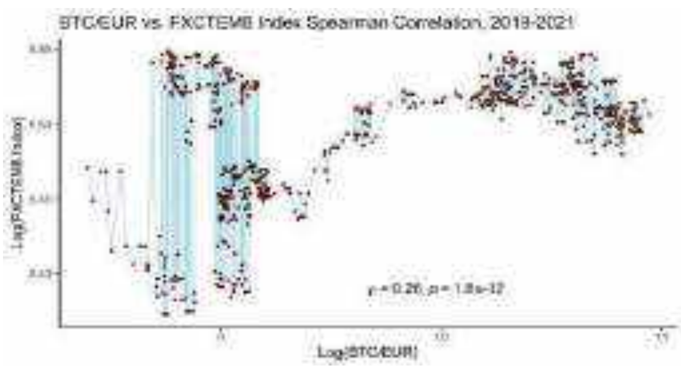
All the correlation coefficients are statistically significant and lower than the coefficients of BTC/EUR vs. EUR/USD. Linearity is violated (see covariation plot & the graph on the left). In this case, examining Kendall and Spearman correlation tests would be appropriate.
Monotonicity does seemingly not hold in the density distribution graph plotted based on half-yearly observations. Therefore, the results of Kendall and Spearman tests should be evaluated accordingly. On top of being positive, both

rho and tau are less than 0.25, while the former is higher than the latter. Consequently, the result shows that USD/JPY is not a successful indicator of the BTC returns.

Correlation with the FXCTEM8 Index

Test	Variable	Statistic	P-value	Sample Estimate
Pearson's product-moment correlation	BTC/EUR ~ FXCTEM8	t = 14.896	2.2e-16	0.482
Kendall's rank correlation tau	BTC/EUR ~ FXCTEM8	z = 7.8476	4.24e-5	0.193
Spearman's rank correlation rho	BTC/EUR ~ FXCTEM8	S = 49233370	1.822e-12	0.256



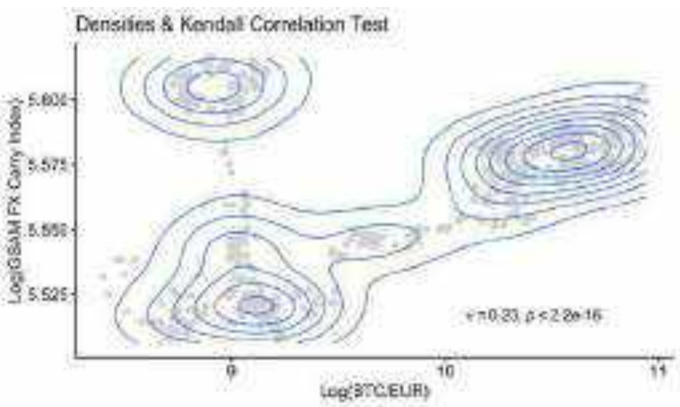


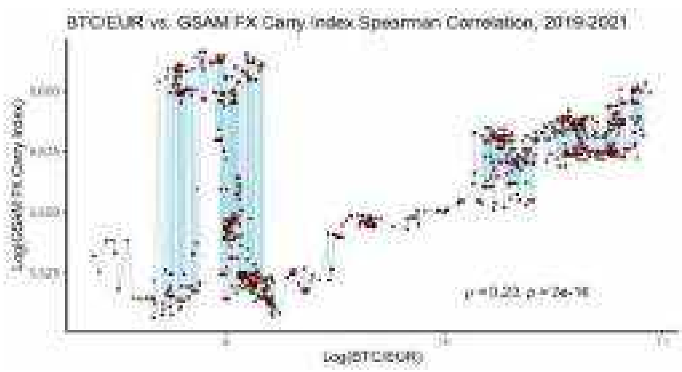
The tests results are statistically significant. Pearson's coefficient R is considerably higher than the others. Although one can tend to interpret this result as a linear relationship between BTC/EUR and FXTEM8 Index prices; since the linearity and normality assumptions are violated, it is not suggestive conclude a linear relationship.

For the Kendall and Spearman tests, similar results are obtained as the USD/JPY correlation tests. Only continuity holds among the assumptions. Both tau and rho are relatively small, so their positive sign does not suggest high level of rank-based correlation. It can be deducted from the correlation results that, price movements of the eight emerging markets included in the index noticeably differs from the BTC/EUR price movements.

Correlation with the GSAM FX Carry

Test	Variable	Statistic	P-value	Sample
Pearson'sproduct-moment	BTC/EUR ~ GSAM FX Carry	t = 10.734	< 2.2e-16	0.482
Kendall's rank correlation tau	BTC/EUR ~ GSAM FX Carry	z = 9.2782	<2.2e-6	0.228
Spearman's rank correlation rho	BTC/EUR ~ GSAM FX Carry	S = 50828888	1.964e-10	0.231

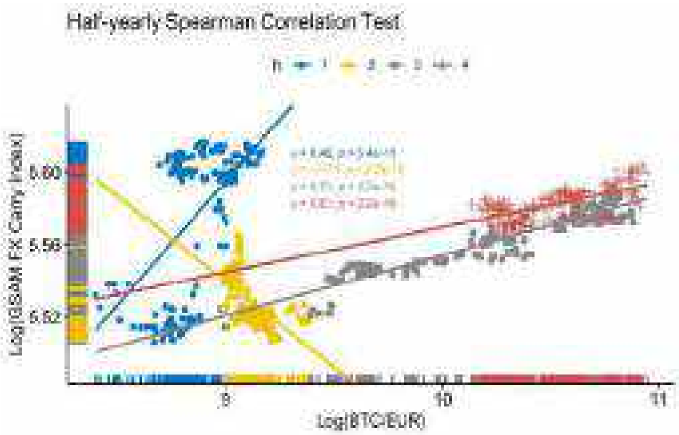




The tests results are statistically significant.

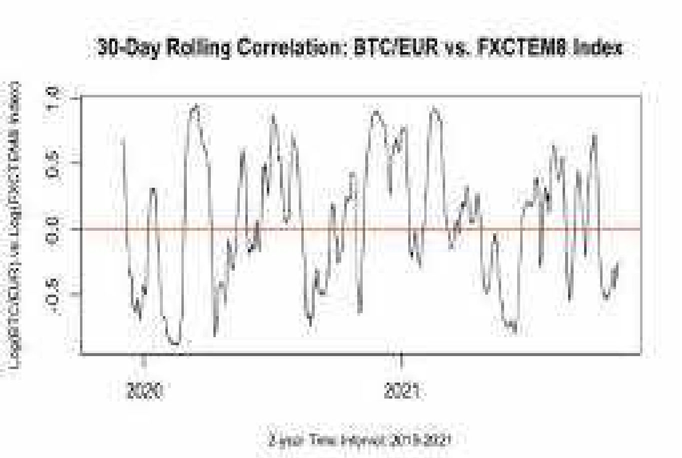
Since not all the assumptions hold for the Pearson test (linearity, normality etc.), Kendall and Pearson test will be taken into account. The relatively high R coefficient (0.48) does not guarantee a linear relationship between BTC/EUR and GSAM FX Carry. Different than the former variables, tau and rho are equal (0.23) in the BTC/EUR vs GSAM FX Carry case. The reason behind this equality can be interpreted as the lack or insignificance of tied ranks. Although the coefficients do not imply a strong association between the variables, there exists a rank-based relationship to some extent.

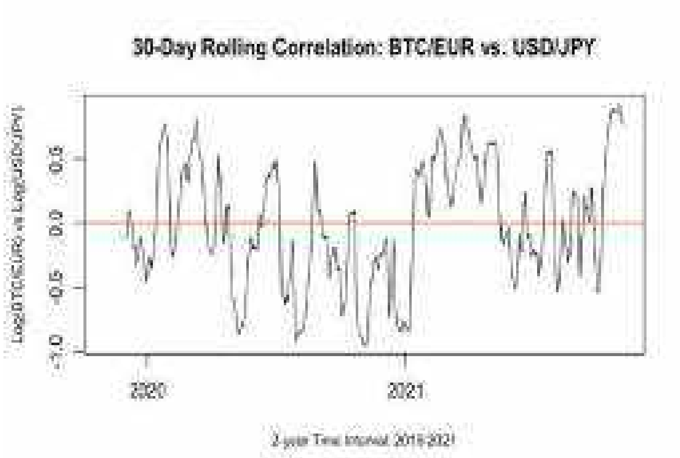
The half-yearly calculated Spearman rho's for BTC/EUR vs. GSAM FX Carry Index are plotted on the right side. Rho for the entire horizon is calculated as 0.23. All the half year correlations, whether negative or positive, are significantly higher than than 0.23 in absolute value reaching a striking level of 0.91 in the third half year. In conclusion, To exploit the relationship between for BTC/EUR vs. GSAM FX Carry Index, sub-intervals might be considered rather than the whole interval



30-Day Rolling Correlations

Rolling correlations are calculated based on Pearson method. Although several assumptions do not hold, to compare the behavior of the rolling correlations among the pairs, the assumptions are considered to be satisfied in this section.





As it can be observed from the graphs, 30-day rolling correlation coefficient between BTC/EUR and FXCTEM8 Index almost reaches to 1 time to time. When the variable FXCTEM8 Index is replaced with USD/JPY, the coefficient starts to reach the approximate level of -1. Consequently, the relevant 30-day correlations display similar strength over the horizon, yet the opposite directions at the peak points. Taking the fact that FXCTEM8 Index follows 8 emerging country currencies into account, the counter movements to BTC/EUR vs. USD/JPY are expected as a result of the foreign exchange relationship between the developed and emerging countries.

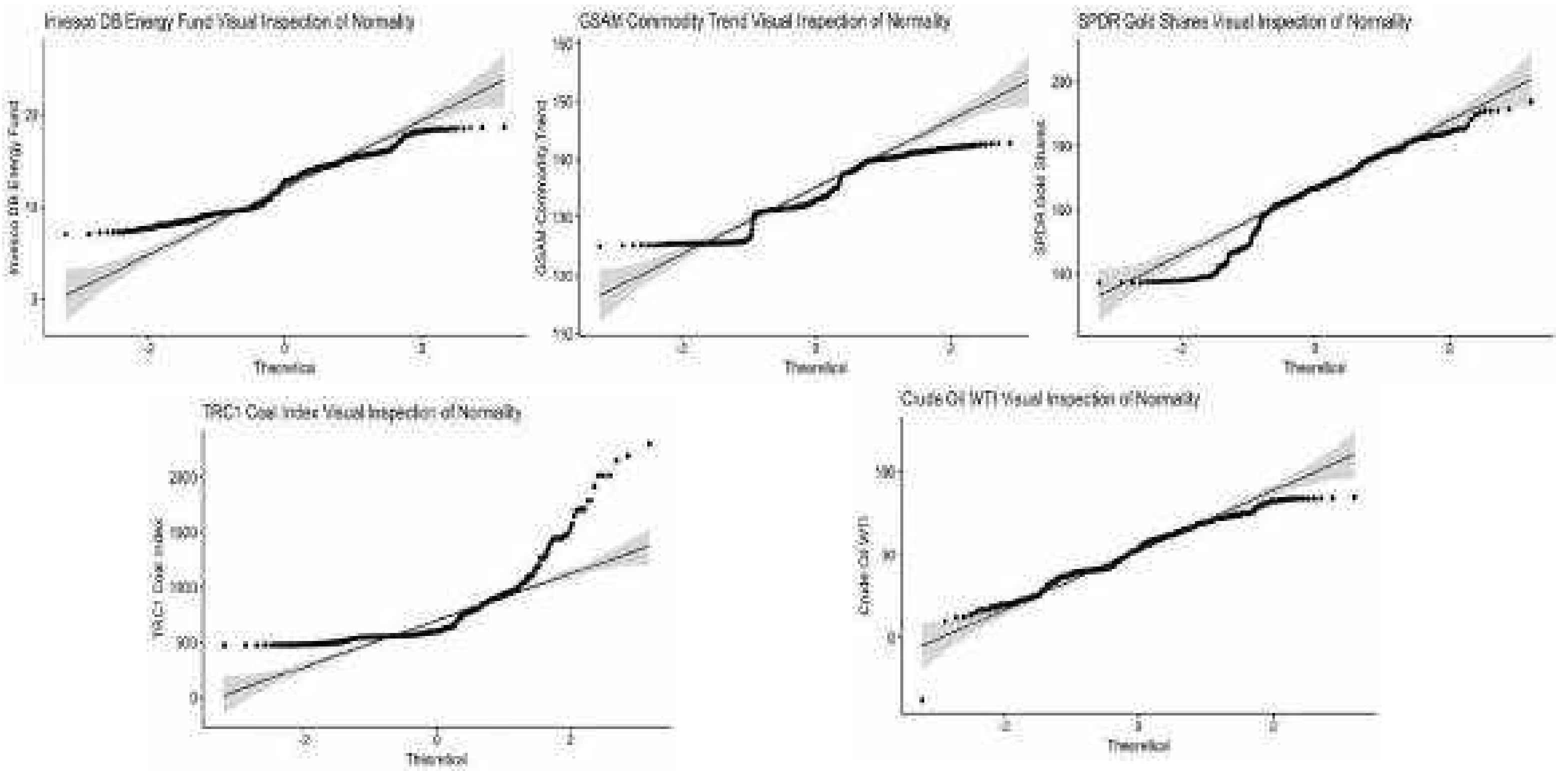
Commodities

Normality tests

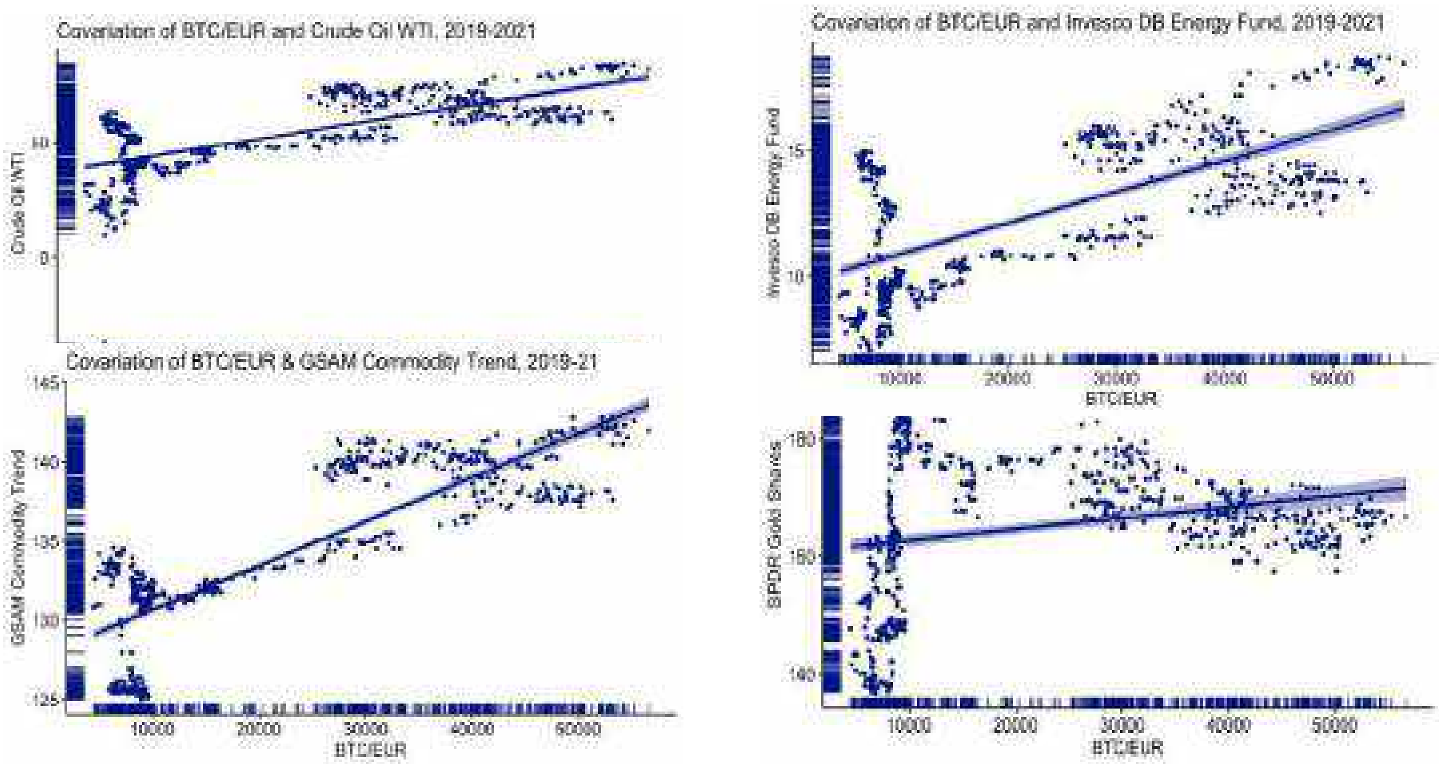
Test	Variable	Statistic	P-value
Shapiro-Wilk normality test	SPDR Gold Shares	W = 0.95073	5.964e-15
Shapiro-Wilk normality test	TRC1 Coal Index	W = 0.74775	< 2.2e-16

Shapiro-Wilk normality test	Crude Oil WTI	W = 0.97322	2.384e-10
Shapiro-Wilk normality test	Invesco DB Energy Fund	W = 0.9511	6.906e-15
Shapiro-Wilk normality test	GSAM Commodity Trend	W = 0.92508	< 2.2e-16

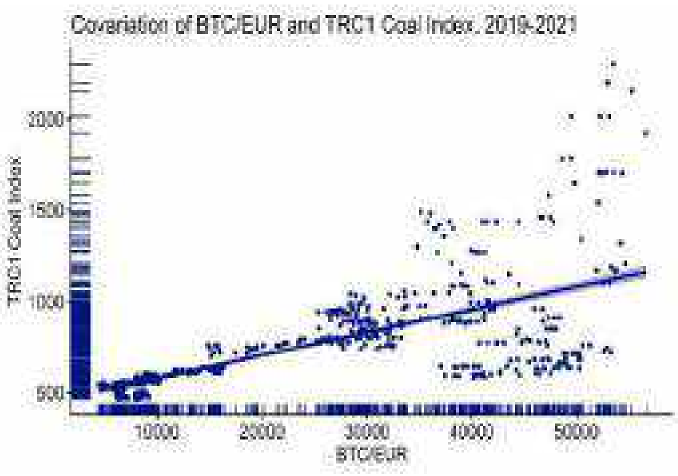
On top of the p-values that are lower than 0.01, the visual inspection plot of normality supports that BTC/EUR and the variables SPDR Gold Shares, TRC1 Coal Index, Crude Oil WTI, Invesco DB Energy Fund GSAM Commodity Trend are not normally distributed.



Linearity Tests

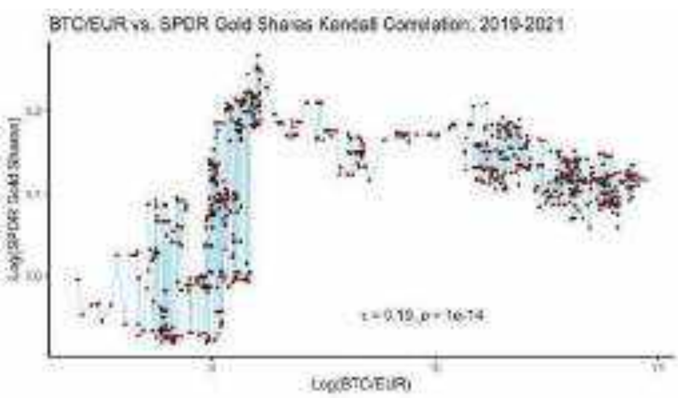


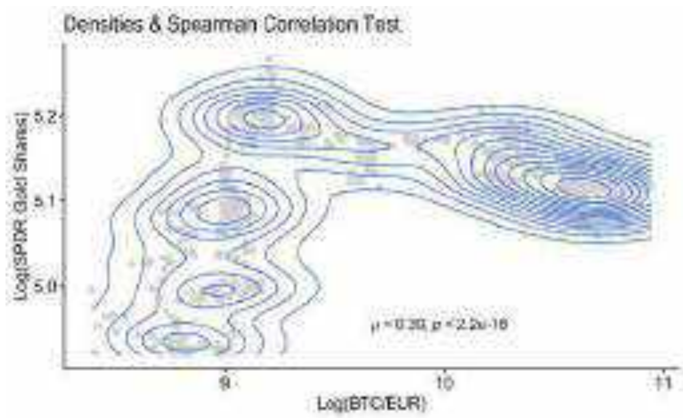
The covariation graphs of BTC/EUR and SPDR Gold Shares, TRC1 Coal Index, Crude Oil WTI, Invesco DB Energy Fund GSAM Commodity Trend are plotted to confirm linearity. Linearity partially holds in the Crude Oil WTI and TRC1 Coal Index plots, in the others linearity does not hold.



Correlation with the SPDR Gold Shares

Test	Variable	Statistic	P-value	Sample Estimate
Pearson's product-moment correlation	BTC/EUR ~ SPDR Gold	t = 10.873	< 2.2e-16	0.372
Kendall's rank correlation tau	BTC/EUR ~ SPDR Gold	z = 7.7381	1.009e-14	0.190
Spearman's rank correlation rho	BTC/EUR ~ SPDR Gold	S = 40665950	< 2.2e-16	0.385

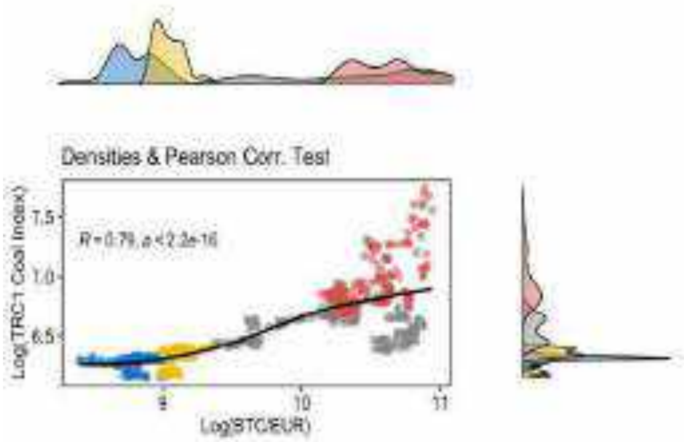


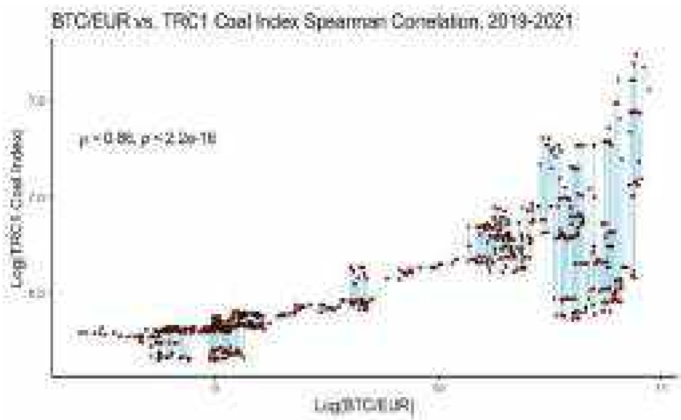


Due the level of the p-values, the null hypothesis can be rejected in all the three tests. Considering the assumptions that do not hold, Kendall and Spearman tests will be explored. Based on tau (0.19) and rho (0.39), the direction of the relationship is positive, with a low strength.
Gold as seen as relatively safe means of investment, while the Bitcoin is considered to be risky. Therefore, the demand for Bitcoin or gold are affected by the level of risk aversion in the markets. Hence, the low level of correlation is consistent. Additionally, there are five concentration points in the density graph of BTC/EUR and SPDR Gold. The rate of the prices does not stay constant throughout the horizon, probably originating from the increasing appetite for BTC.

Correlation with the TRC1 Coal Index

Test	Variable	Statistic	P-value	Sample Estimate
Pearson's product-moment correlation	BTC/EUR ~ TRC1 Coal	t = 35.162	< 2.2e-16	0.792
Kendall's rank correlation tau	BTC/EUR ~ TRC1 Coal	z = 26.791	< 2.2e-16	0.661
Spearman's rank correlation rho	BTC/EUR ~ TRC1 Coal	S = 9453004	< 2.2e-16	0.857

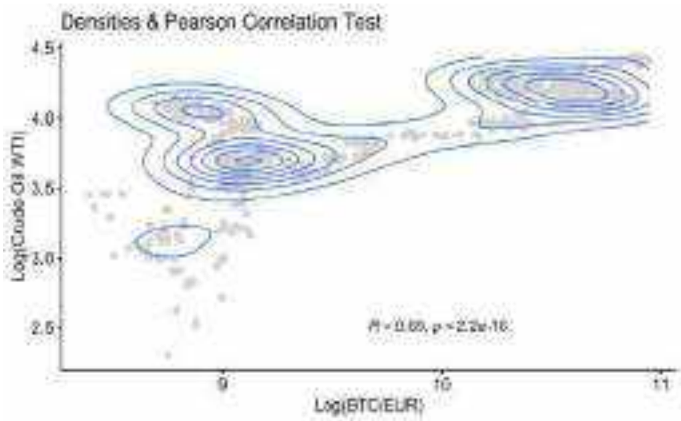


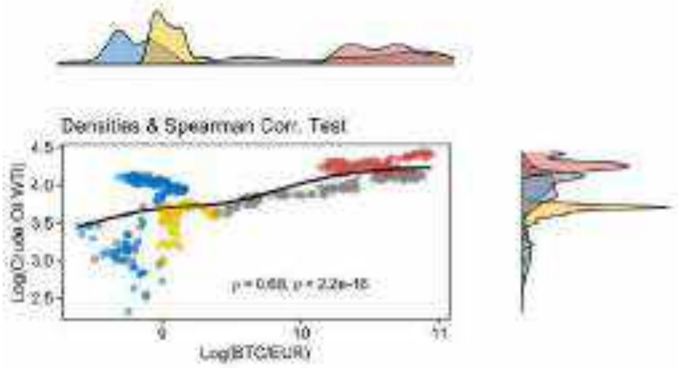


Though some of the assumptions such as normality are violated, since there is a partial linearity between the variables the Pearson test can add up value to the analysis. R is equal to 0.79 that suggests strong linear correlation between BTC/EUR and TRC1 Coal Index returns. Additionally, the association gets stronger when the rank-based correlation test is applied. It is further bolstered by the high level of Spearman's rho, 0.86. Coal futures returns seem significantly correlate to BTC returns.

Correlation with the Crude Oil WTI

Test	Variable	Statistic	P-value	Sample Estimate
Pearson's product-moment correlation	BTC/EUR ~ CL1 (WTI)	t = 26.062	< 2.2e-16	0.693
Kendall's rank correlation tau	BTC/EUR ~ CL1 (WTI)	z = 18.881	< 2.2e-16	0.466
Spearman's rank correlation rho	BTC/EUR ~ CL1 (WTI)	S = 21036434	< 2.2e-16	0.680



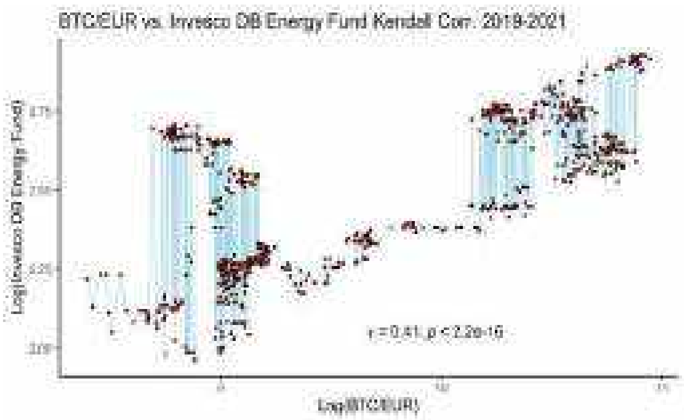


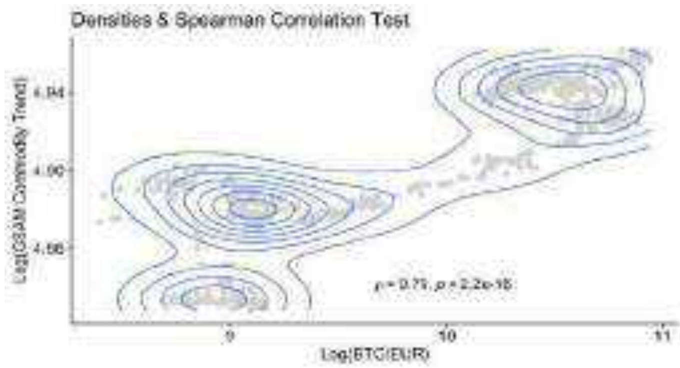
Before the tests are conducted, the outlier observation, in which Crude Oil WTI priced negatively, is omitted. Due to the same reason, Pearson test is valuable to evaluate the linear relationship between BTC/EUR and Crude Oil WTI. The coefficient R is around 0.69, supporting a significant linear relationship. Furthermore, the Spearman's rho (0.68) corroborates meaningful association between the variables.

Correlation with the Invesco DB Energy Fund & GSAM Commodity Trend

Test	Variable	Statistic	P-value	Sample Estimate
Pearson's product-moment correlation	BTC/EUR ~ Invesco DB	t = 23.141	< 2.2e-16	0.649
Kendall's rank correlation tau	BTC/EUR ~ Invesco DB	z = 16.618	< 2.2e-16	0.410
Spearman's rank correlation rho	BTC/EUR ~ Invesco DB	S = 29583547	< 2.2e-16	0.552

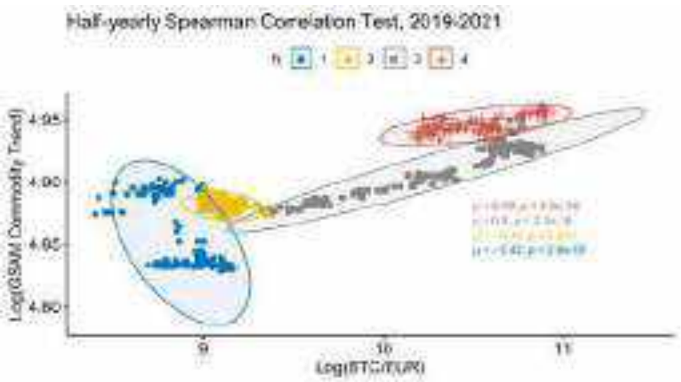
Test	Variable			Statistic	P-value	Sample Estimate
Pearson's product-moment correlation	BTC/EUR	~	GSAM	t = 44.818	< 2.2e-16	0.855
Kendall's rank correlation tau	BTC/EUR	~	GSAM	z = 23.316,	< 2.2e-16	0.575
Spearman's rank correlation rho	BTC/EUR	~	GSAM	S = 14121000	< 2.2e-16	0.7861

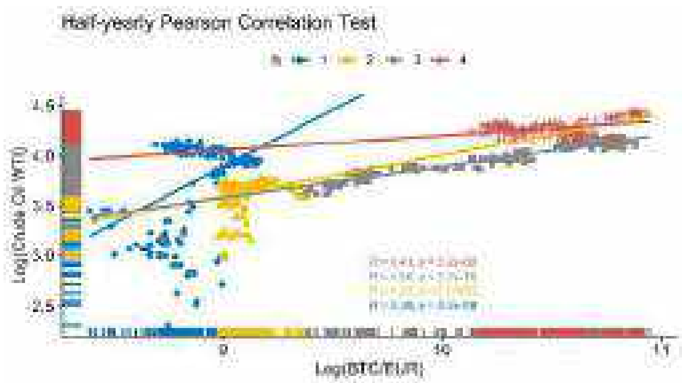




The assumption of linearity fails in addition to normality and monotonicity. Kendall and Spearman correlation tests suggest moderate rank-based correlation (tau is equal to 0.41 & rho is 0.55). The economic interpretation of the tests between BTC/EUR and the Invesco DB Energy Fund are in line with the other energy commodities above.

The assumption of linearity fails in addition to normality and monotonicity. Spearman's rho is quite a high level of 0.79. GSAM Commodity Trend follows Metals, Energy and Ags/Softs Sector Trend strategies. The strong relatonship may be interpreted as Bitcoin price incorproates commodities.



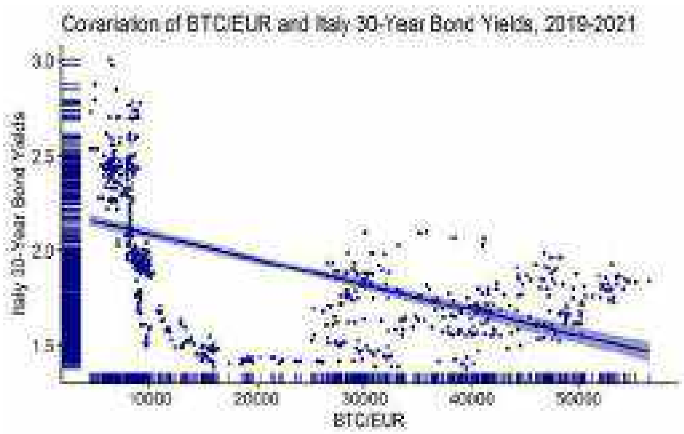


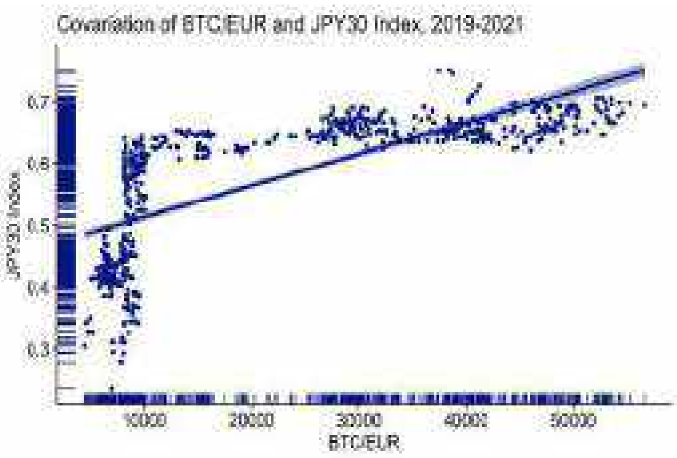
Half-yearly correlation plots of BTC/EUR against GSAM Commodity Trend and Crude Oil WTI yielded interesting results. On the left side above, the Spearman's rho is calculated as 0.9 in the third half year, while Pearson's R is 0.97 in the graph located on the right for the same period. The period is characterized by almost perfect correlation, and this can be attributed the market conditions after the first shock of Covid-19 pandemic has passed. The production levels started to increase that raised the need for commodities and oil as inputs. Also, risky assets such as Bitcoin attracted more attention.

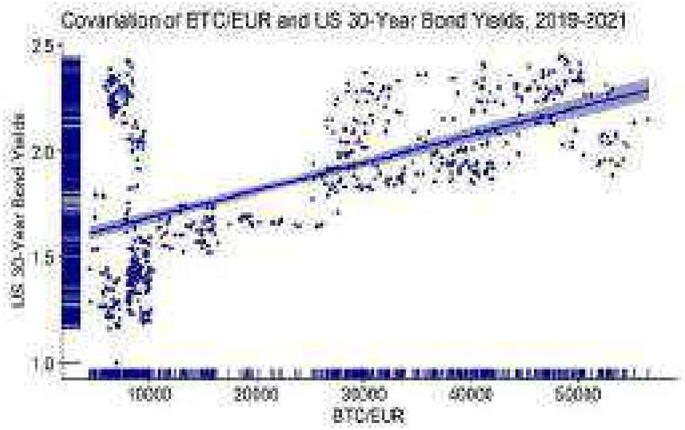
Bonds

Linearity Tests

The covariation graphs of BTC/EUR and IT30, JPY30, US30 are plotted in order to assess linearity assumption. The condition does not hold definitely for all the variables based on the covariation plots.

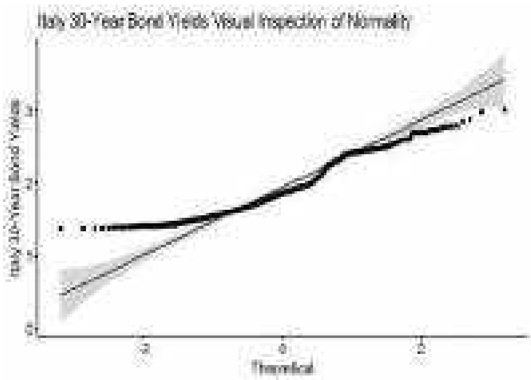


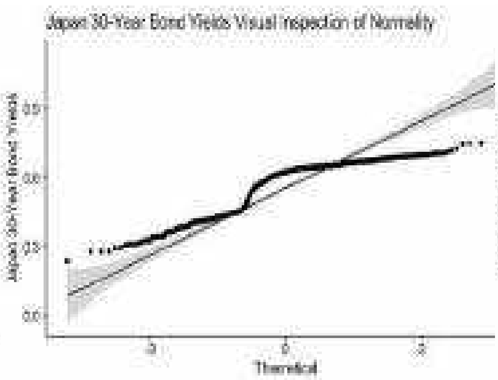


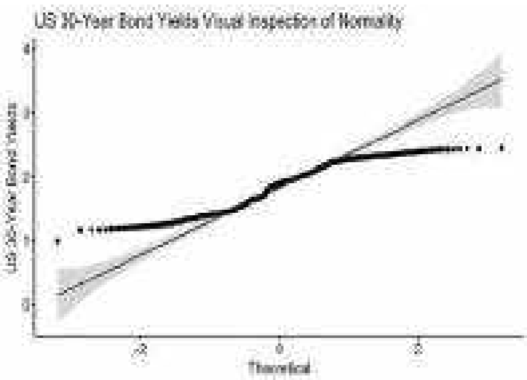


Normality tests

Test	Variable	Statistic	P-value
Shapiro-Wilk normality	BTC/EUR	W = 0.83207	< 2.2e-16
Shapiro-Wilk normality	IT30	W = 0.94167	< 2.2e-16
Shapiro-Wilk normality	JPY30	W = 0.86159	< 2.2e-16
Shapiro-Wilk normality	US30	W = 0.93884	2.384e-10







According to the graphs above and normality tests, all the variables are not normally distributed.

Correlation with the 30-Year Bond

Test	Variable	Statistic	P-value	Sample Estimate

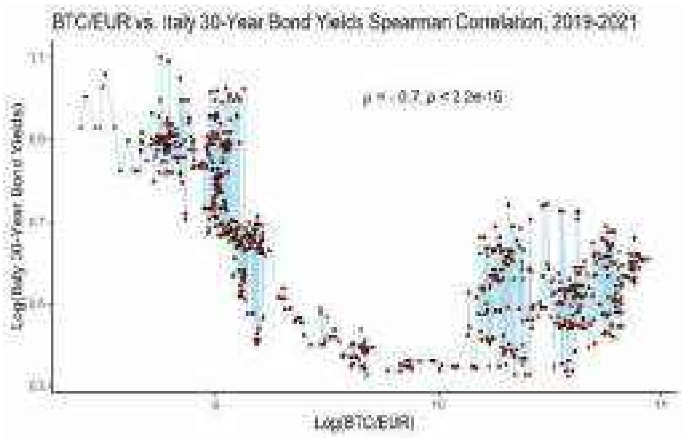
Pearson's product-moment correlation	BTC/EUR ~ IT30	t = -24.318	< 2.2e-16	-0.668
Kendall's rank correlation tau	BTC/EUR ~ IT30	z = -19.662	< 2.2e-16	-0.485
Spearman's rank correlation rho	BTC/EUR ~ IT30	S= 12623393	< 2.2e-16	-0.701

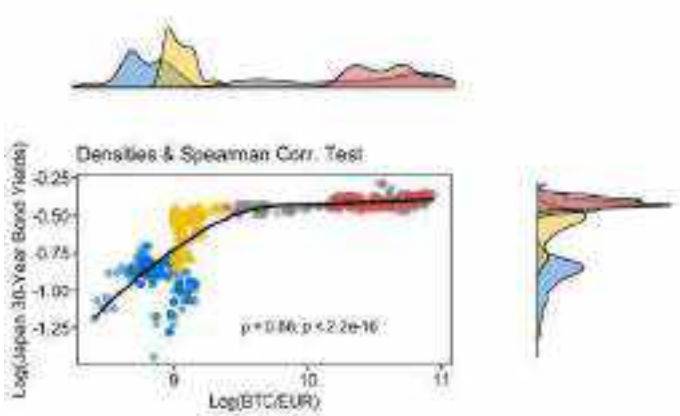
Test	Variable	Statistic	P-value	Sample Estimate
Pearson's product-moment correlation	BTC/EUR ~ JPY30	t = 31.802	< 2.2e-16	0.761
Kendall's rank correlation tau	BTC/EUR ~ JPY30	z = 27.575	< 2.2e-16	0.681
Spearman's rank correlation rho	BTC/EUR ~ JPY30	S = 8092991	< 2.2e-16	0.877

Test	Variable	Statistic	P-value	Sample Estimate
Pearson's product-moment correlation	BTC/EUR ~ US30	t = 17.375	< 2.2e-16	0.540
Kendall's rank correlation tau	BTC/EUR ~ US30	z = 14.422	< 2.2e-16	0.355
Spearman's rank correlation rho	BTC/EUR ~ US30	S = 38520583	< 2.2e-16	0.417

All of the tests have a p-value < 2.2e-16 meaning that, all of them are statistically significant at the 99% confidence level. Linearity tests and its graphs plainly demonstrate that linearity assumption does not hold; hence, instead of Pearson test, Spearman and Kendall tests should be considered for statistically meaningful results.

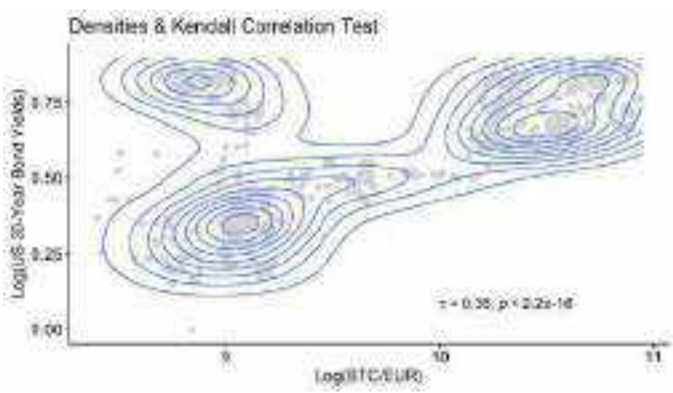
Relationship between BTC/EUR and IT30 resulted in a Spearman's rho of -0.70, which illustrated on the left graph, indicating a strong negative correlation between variables.



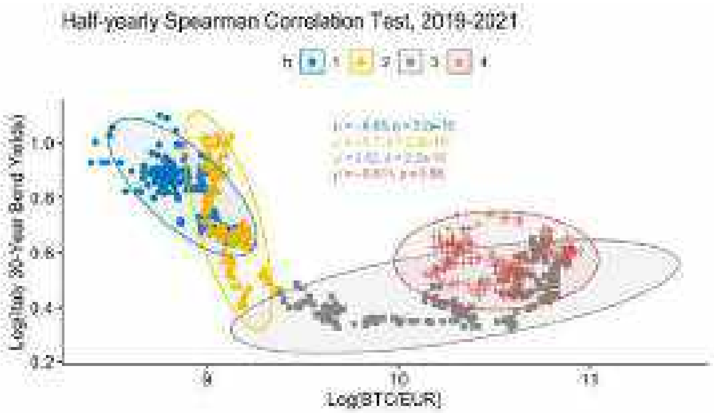


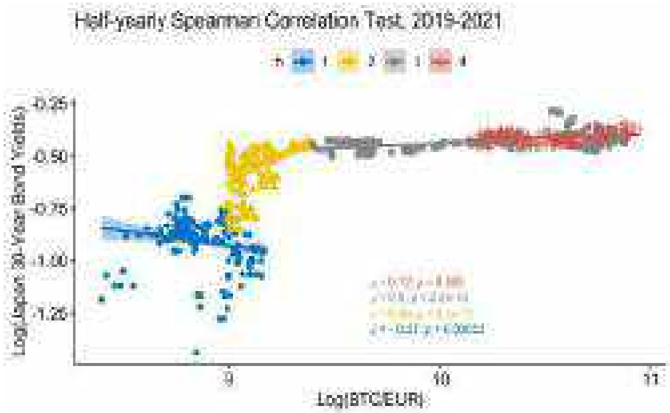
In contrast to BTC/EUR and 30-year bond yields of Italy, there is a strong positive correlation among BTC/EUR and JPY30 that can apparently be observed by 0.88 Spearman's rho.

Lastly, one may recognize a moderate level positive relationship between BTC/EUR and US30. If Kendall's rank correlation tau test is looked at to quantify it, 0.36 sample estimate suggests that strength of the relationship is neither very high nor very low.



Half-yearly Correlations



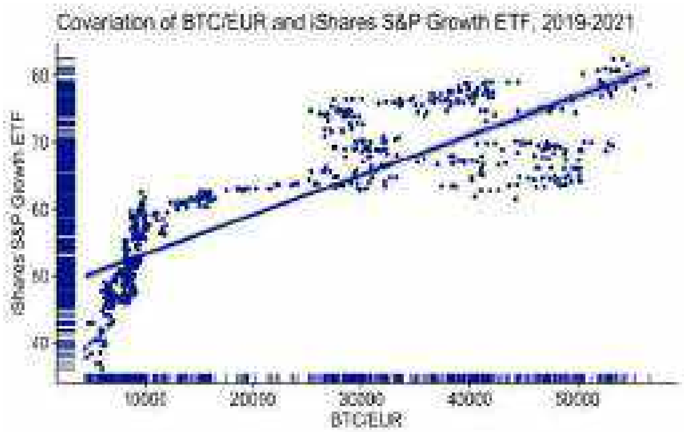


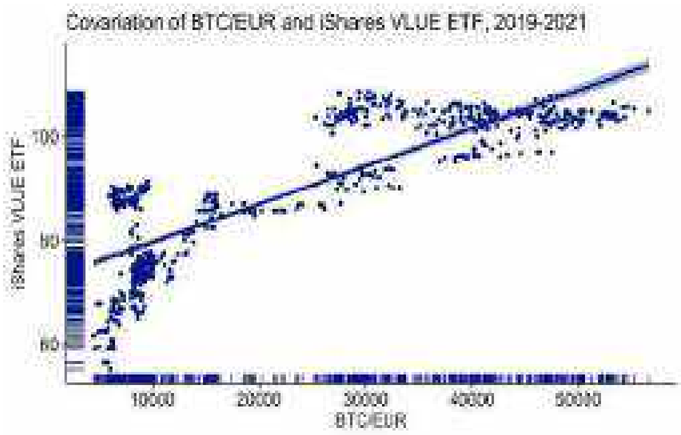
6-month correlation tests of BTC/EUR against IT30 and JPY30 produced interesting outcomes. BTC/EUR vs. IT30 graph plotted on the left side above demonstrated that besides the third 6-month period, which has a 0.62 rho, every other 6-month period resulted in a negative correlation with the last half not being statistically significant, reaching peak during the second half year. On the contrary, other than resulting in a negative correlation with BTC/EUR in the first half year which includes the months of initial COVID-19 pandemic acceleration, 30-year bond yields of Japan had positive correlation, noticeably positive during the second and third half years, with BTC/EUR. It is interesting to observe such a considerable positive relationship, highest among the comparisons, during the second 6-month period, when COVID-19 pandemic has still been surging for several countries. According to data, one can interpret that Japan is one of the fastest countries recovering from the pandemic.

Equities

Linearity Tests

The observations of BTC/EUR against iShares MSCI USA Value Factor ETF and IVW US Equity do not follow a linear path.

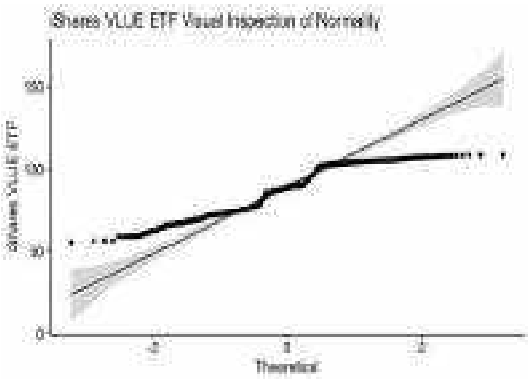


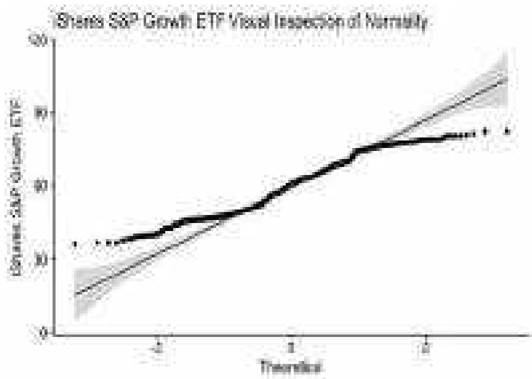


Normality tests

Test	Variable	Statistic	P-value
Shapiro-Wilk normality test	iShares MSCI USA Value	W = 0.92618	2.2e-16
Shapiro-Wilk normality test	IVW US Equity	W = 0.96598	4.844e-12

With the p-values much smaller than 0.01 and the visual checks, the normality for both variable is rejected.

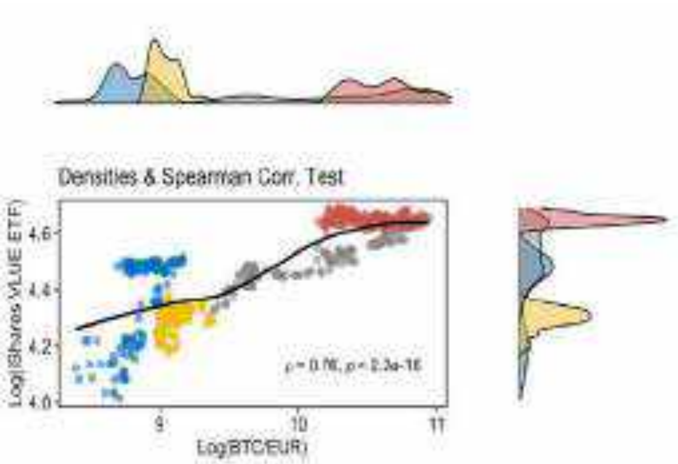


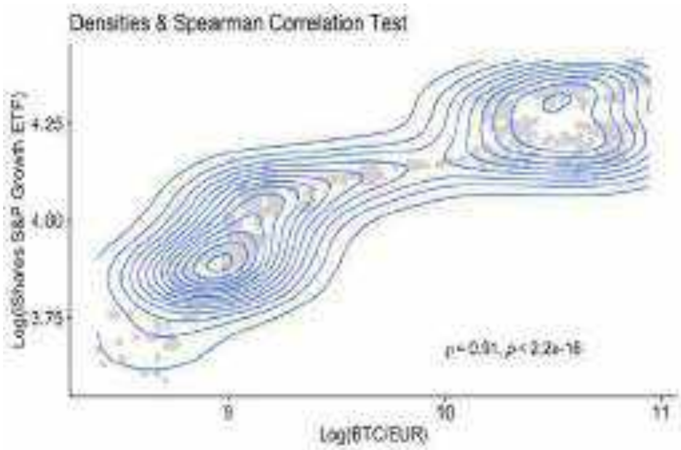


Correlation with the iShares MSCI USA Value Factor ETF and IVW US Equity

Test	Variable	Statistic	P-value	Sample Estimate
Pearson's product-moment correlation	BTC/EUR ~ iShares	t = 40.609	< 2.2e-16	0.832
Kendall's rank correlation tau	BTC/EUR ~ iShares	z = 22.175	< 2.2e-16	0.547
Spearman's rank correlation rho	BTC/EUR ~ iShares	S = 15772648	< 2.2e-16	0.761

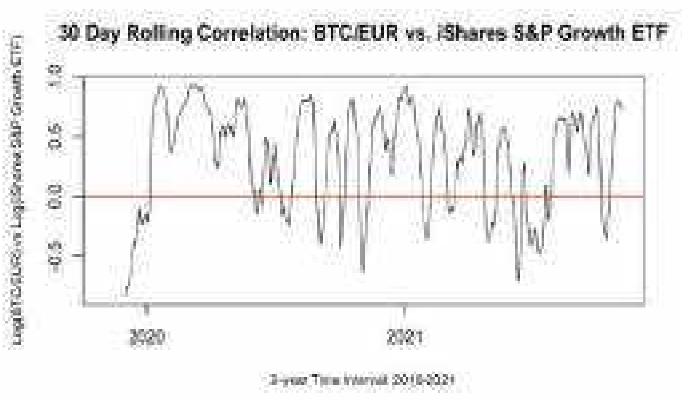
Test	Variable	Statistic	P-value	Sample Estimate
Pearson's product-moment correlation	BTC/EUR ~ IVW US	t = 56.56	< 2.2e-16	0.901
Kendall's rank correlation tau	BTC/EUR ~ IVW US	z = 30.174	< 2.2e-16	0.744
Spearman's rank correlation rho	BTC/EUR ~ IVW US	S = 5669532	< 2.2e-16	0.914





Considering that both linearity and normality test do not hold for the variables, Kendall and Spearman tests should be considered. P-values for all tests are statistically significant at the 99% confidence level. With Spearman's rho being 0.76, relationship between BTC/EUR and iShares MSCI USA Value Factor ETF can be understood as strongly positive. This might be from the fact that iShares MSCI USA Value Factor ETF tracks the companies that have lower valuations based on their fundamentals meaning that, even though they include potential growth opportunities, process still includes certain risk factors compared to investing on very large companies. Knowing that Bitcoin also perceived as riskier investment by the market, high positive correlations of these variables can be expected.

Same conceptual understanding and relationship can be applicable also for the correlation results between BTC/EUR and iShares S&P Growth ETF, which tracks the companies that exhibit growth characteristics. In fact, sign and the strength of their relationship is even higher –Spearman's rho equals 0.91- than the previous case. As risk appetite of the market increases, both the demand for Bitcoin and companies that contain growth opportunities increases; opposite is true also.



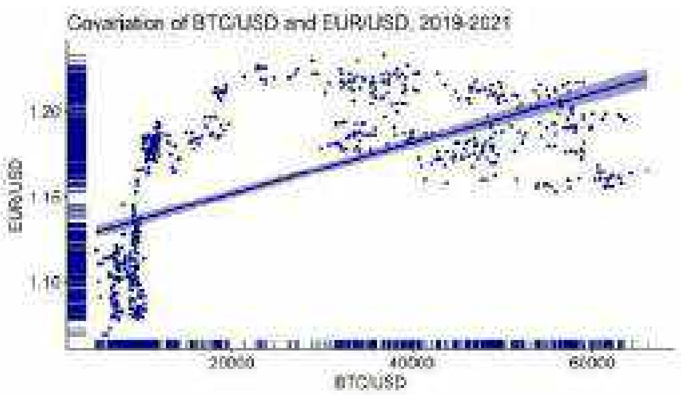
30-Day rolling correlations of BTC/EUR against iShares MSCI USA Value Factor ETF and iShares S&P Growth ETF yielded some interesting results. It is noticeable that throughout the horizon, 30-day correlations of both pairs mostly had positive sign which, even reached a near perfect correlation for some periods, is a demonstration of their strong positive correlation overall.

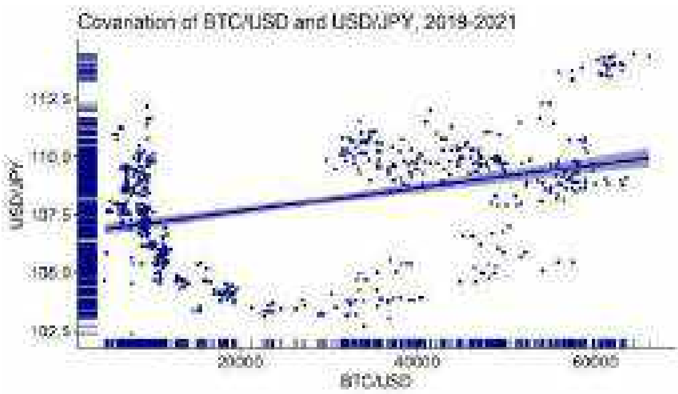
BTC/USD

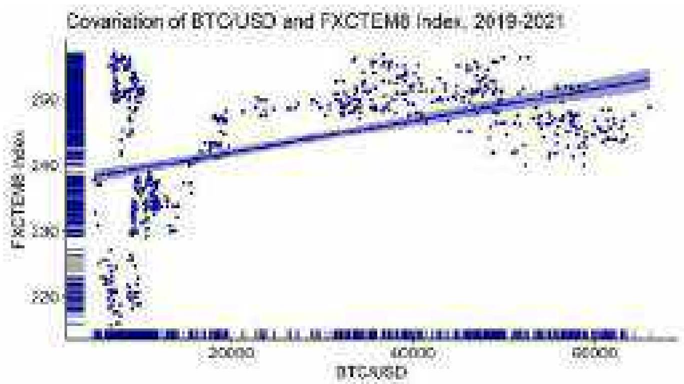
FX

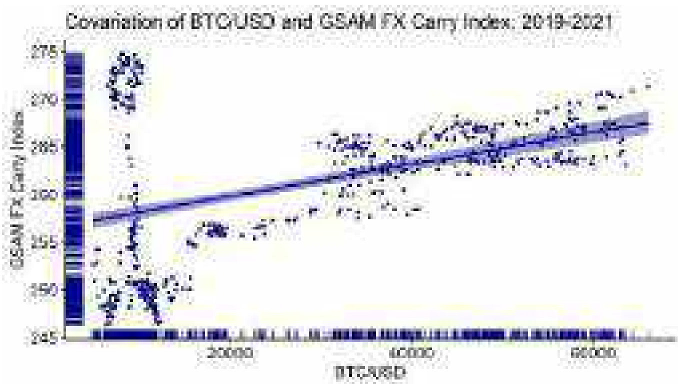
Linearity Tests

The covariation graphs of BTC/USD and EUR/USD, USD/JPY, FXCTEM8 Index, GSAM FX Carry are plotted to visualize the nature of the relationships. As clearly can be seen in the graphs above, the relationship between the prices of BTC/USD and the variables lacked linearity in the analysis horizon. On top of not following a fitted line, the observations also fall around the line in with irregular patterns.





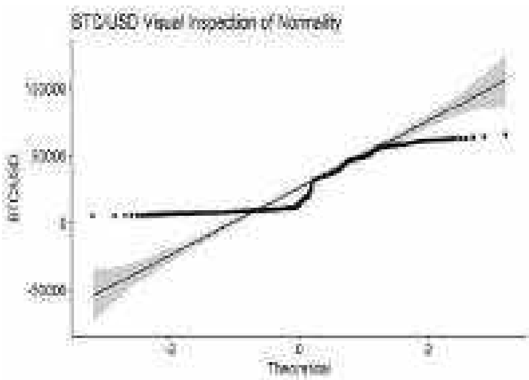




Normality tests

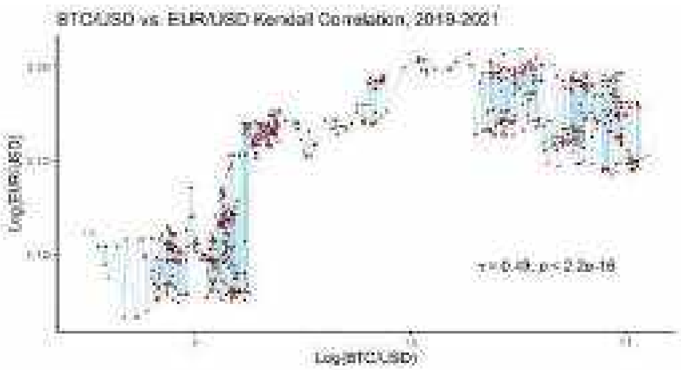
Test	Variable	Statistic	P-value
Shapiro-Wilk normality test	BTC/USD	W = 0.83317	< 2.2e-16

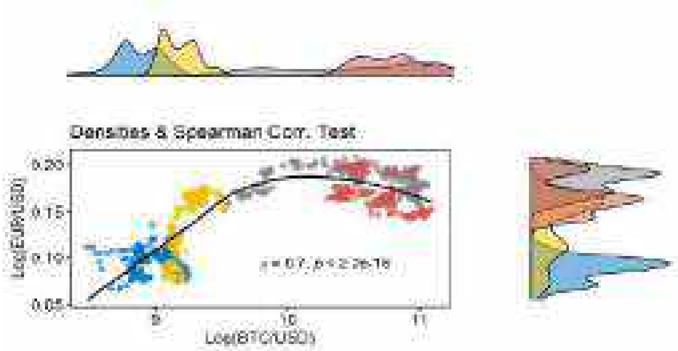
Shapiro-Wilk normality test is applied in all the cases, the test concludes in a p-value much smaller than 0.01. BTC/USD and EUR/USD, USD/JPY, FXCTEM8 Index, GSAM FX Carry are not normally distributed in the analysis period (see previous section for test result and QQ plots).



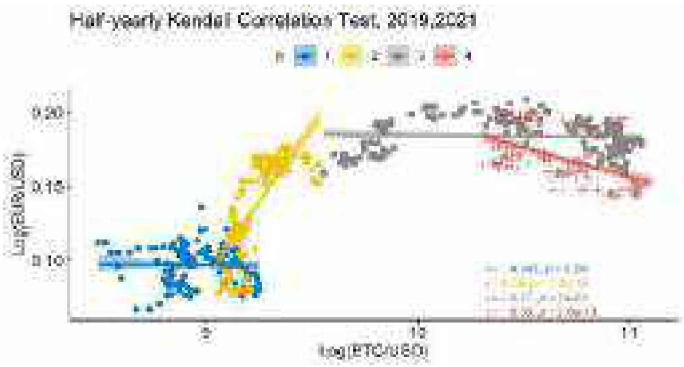
Correlation with the EUR/USD

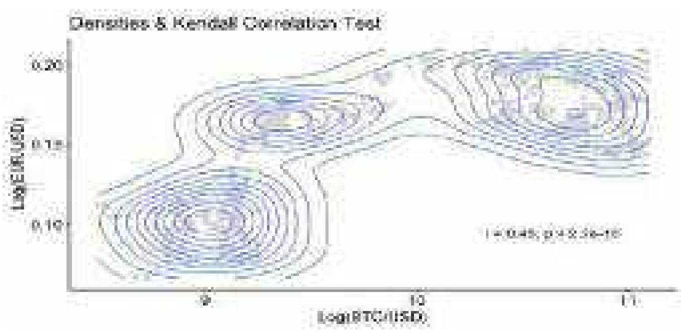
Test	Variable	Statistic	P-value	Sample Estimate
Pearson's product-moment correlation	BTC/USD ~ EUR/USD	t = 30.025	2.2e-16	0.742
Kendall's rank correlation tau	BTC/USD ~ EUR/USD	z = 19.692	2.2e-16	0.485
Spearman's rank correlation rho	BTC/USD ~ EUR/USD	S = 19612783	2.2e-16	0.703





In BTC/USD vs. EUR/USD case, several assumptions of the Pearson test do not hold such as normal distribution and linearity. Due to non-normal distributions, logarithmic transformation is applied to the data before the correlation tests. Kendall and Spearman tests would produce results that are statistically more meaningful. As well as the Spearman's rho and Kendall's tau coefficients are positive.



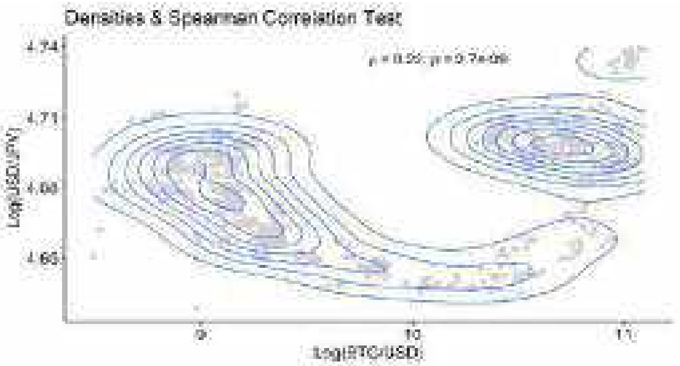


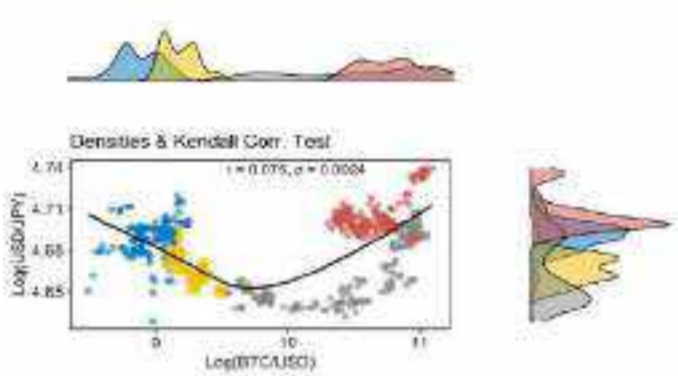
The graph on the left illustrates decomposition of the 6-months periods of the correlations for the BTC/USD vs. EUR/USD. If one would compare the periods, tau is only positive during the second 6-month time frame. 0.56 correlation for this period is an important contributor to overall correlation for the 2-year horizon is 0.49

Three concentration points in the density graph of BTC/USD and EUR/USD are observed. Changing demand for the BTC throughout the horizon might be the cause of fluctuating prices. Considering the test results, which may indicate that BTC/USD shares a similar risk profile with EUR/USD, with a significant wider scale of fluctuations.

Correlation with the USD/JPY

Test	Variable	Statistic	P-value	Sample Estimate
Pearson's product-moment correlation	BTC/USD ~ USD/JPY	t = 7.3835	4.198e-13	0.263
Kendall's rank correlation tau	BTC/USD ~ USD/JPY	z = 3.035	0.002406	0.074
Spearman's rank correlation rho	BTC/USD ~ USD/JPY	S = 51811459	2.742e-09	0.217

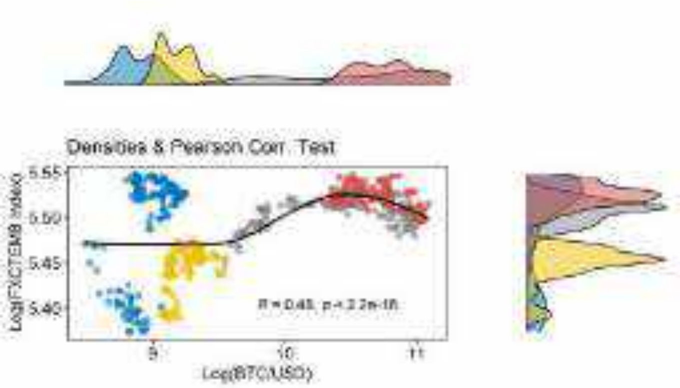


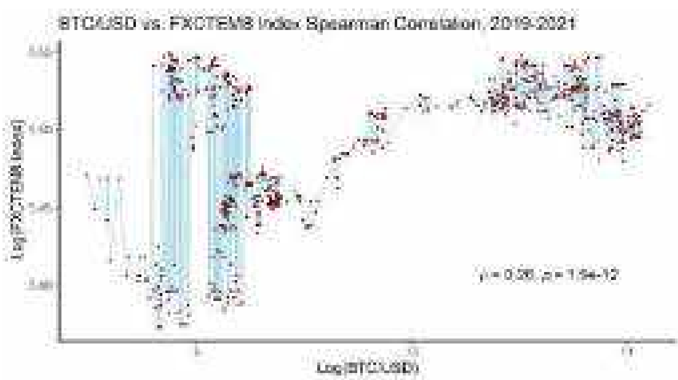


Linearity is violated hence, examining Kendall and Spearman correlation tests would be appropriate. Monotonicity does seemingly not hold in the density distribution graph plotted based on half-yearly observations. Therefore, the results of Kendall and Spearman tests should be evaluated accordingly. Comparing rho and tau; it is evident that rank-based relationships between BTC/USD and USD/JPY is weaker than EUR/USD, implying that USD/JPY is not a successful indicator of the demand for BTC.

Correlation with the FXCTEM8 Index

Test	Variable	Statistic	P-value	Sample Estimate
Pearson's product-moment correlation	BTC/USD ~ FXCTEM8 Index	t = 14.983	2.2e-16	0.484
Kendall's rank correlation tau	BTC/USD ~ FXCTEM8 Index	z = 8.0317	9.612e-16	0.198
Spearman's rank correlation rho	BTC/USD ~ FXCTEM8 Index	S = 49249496	1.915e-12	0.255



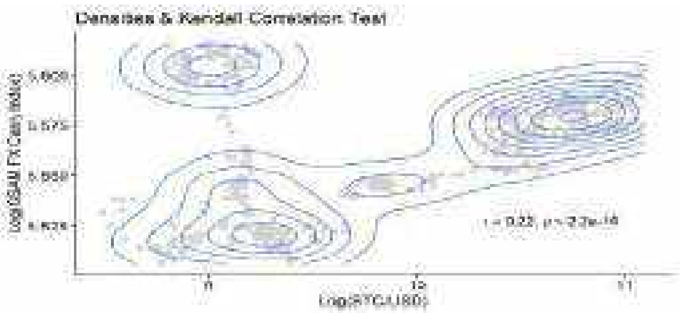


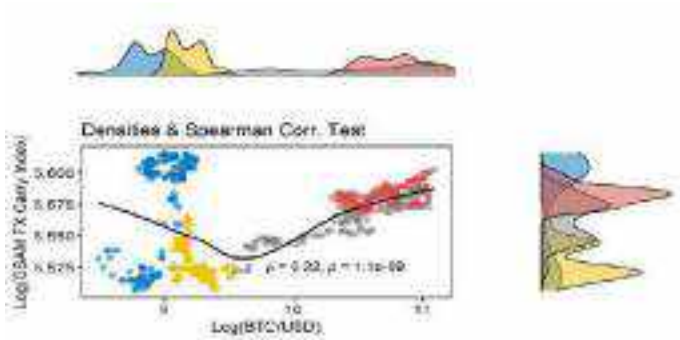
Only continuity holds among the assumptions. Relatively small tau and rho indicate that there is no high level of rank-based correlation. Results suggest that returns are considerably different among the eight emerging markets included in the index and BTC/USD.

Correlation with the GSAM FX Carry

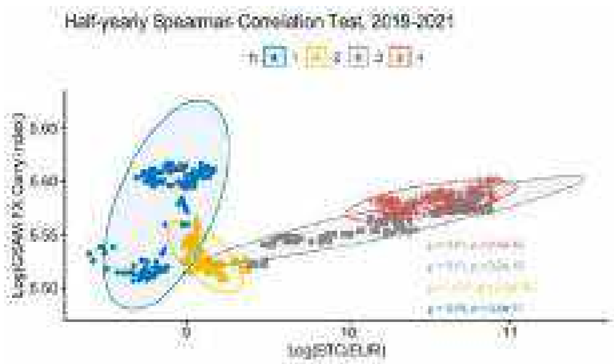
Test	Variable	Statistic	P-value	Sample Estimate
Pearson's product-moment correlation	BTC/USD ~ GSAM FX Carry	t = 10.223	2.2e-16	0.353
Kendall's rank correlation tau	BTC/USD ~ GSAM FX Carry	z = 8.978	2.2e-16	0.221
Spearman's rank correlation rho	BTC/USD ~ GSAM FX Carry	S = 51476645	1.14e-09	0.222

All correlation coefficients are closer to each other compared to previous variables. However, their value in absolute terms do not assert a strong relationship. Linearity does not hold for the analysis.
It can be observed that there are four high-density areas; yet they are mostly concentrated on the far-right side of the graph.



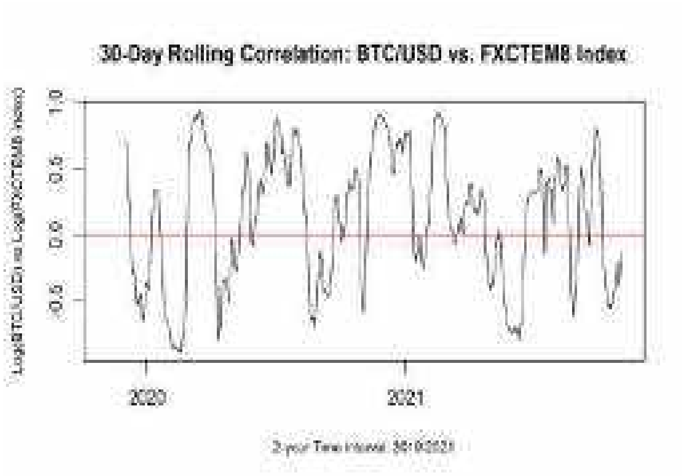


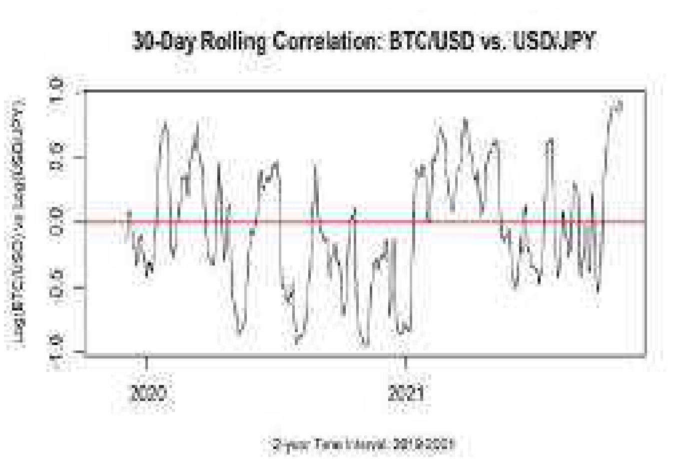
6-month decomposition of Spearman's rho correlations for BTC/USD vs. GSAM FX Carry Index can be seen from the left. Noticeable negative correlation (-0.69) is observed during the second 6-month period. Although significant positive correlations are occurred during the third (0.91) and fourth (0.59) 6-month periods, negative effect of the second period holds the overall correlation for the 2-year period at 0.22.



30-Day Rolling Correlations

Rolling correlations are calculated based on Pearson method. Although several assumptions do not hold, to compare the behavior of the rolling correlations among the pairs, the assumptions are considered to be satisfied in this section Above graphs represent the 30-Day rolling correlations of BTC/USD vs. FXCTEM8 Index and BTC/USD vs USD/JPY exchange rate. It can be observed from the left graph that FXCTEM8 Index, which includes 8 emerging countries, and BTC/USD has certain periods that corresponding correlation almost reached the 1.0; in contrast, USD/JPY and BTC/USD correlation is observed nearly -1.0 for some periods. We can assess that USD/JPY is most consistently negative correlated through the period whereas it is hard to spot patterns in respect to emerging market currencies.

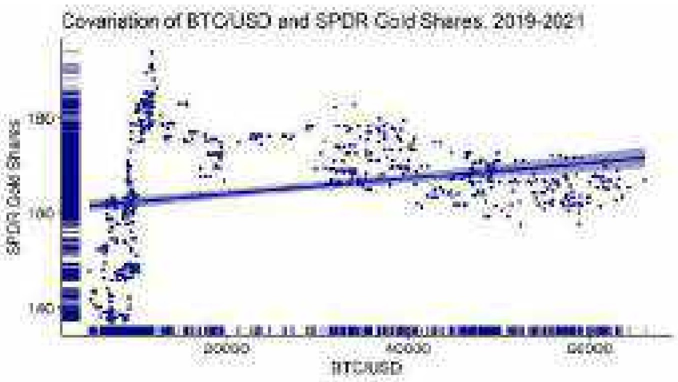


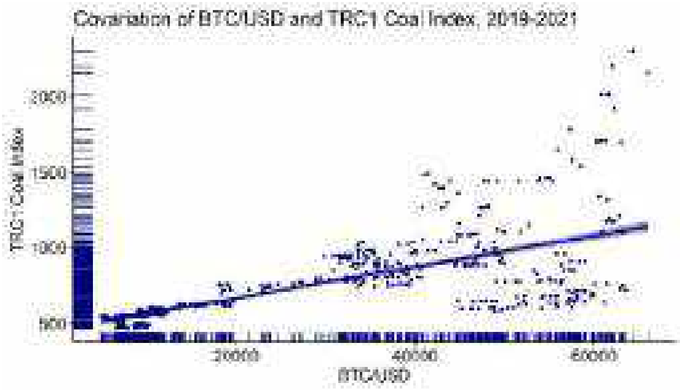


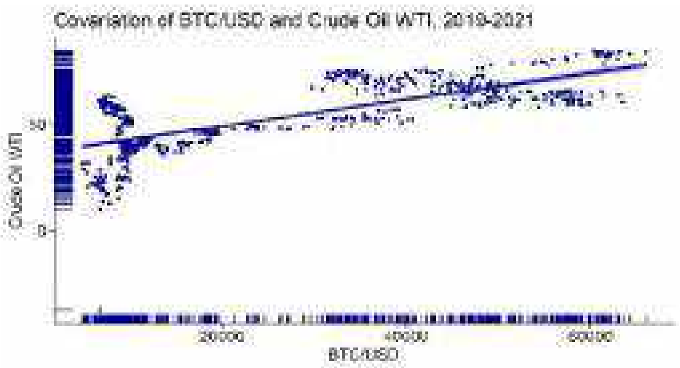
Commodities

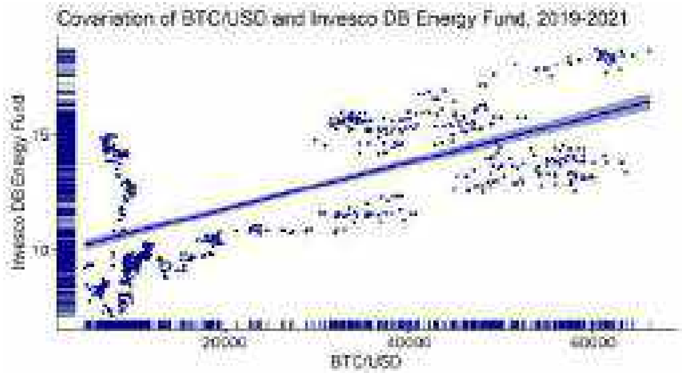
Linearity Tests

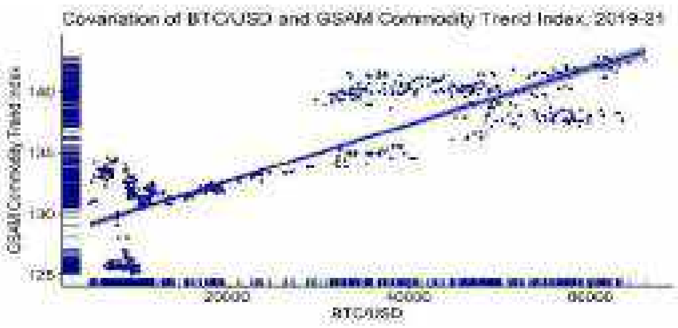
Differently from the previous analysis, here, linearity partially holds in the Crude Oil WTI and TRC1 Coal Index plots; however, others do not satisfy linearity.









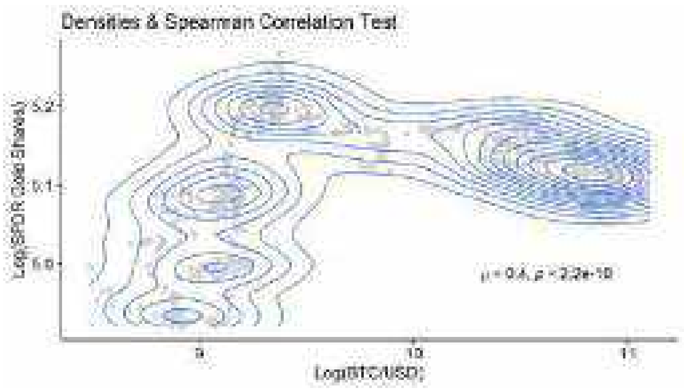


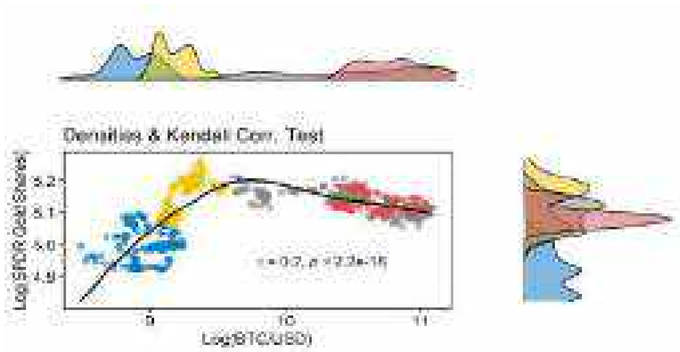
Normality tests

Before applying the three correlation tests (Pearson, Kendall, Spearman), the assumption of normality underlying the tests is checked over BTC/USD (see previous section for test result and QQ plots).

Correlation with the SPRD Gold Shares

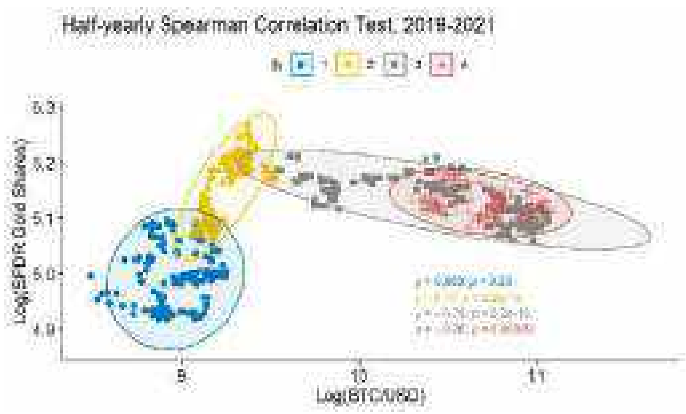
Test	Variable	Statistic	P-value	Sample Estimate
Pearson's product-moment correlation	BTC/USD ~ SPDR Gold Shares	t = 11.636	2.2e-16	0.394
Kendall's rank correlation tau	BTC/USD ~ SPDR Gold Shares	z = 8.2429	2.2e-16	0.203
Spearman's rank correlation rho	BTC/USD ~ SPDR Gold Shares	S = 39505702	2.2e-16	0.403





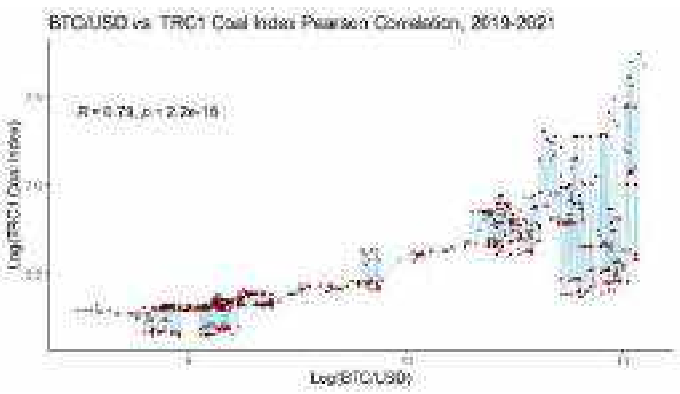
Results of p-values suggest that null hypothesis is rejected for all three of the tests. Due to unsatisfied assumptions of the Pearson test, Spearman and Kendall tests should be investigated instead. Although both rho and tau being positive, their correlation strength with BTC/USD is relatively low (0.4 and 0.2 respectively).

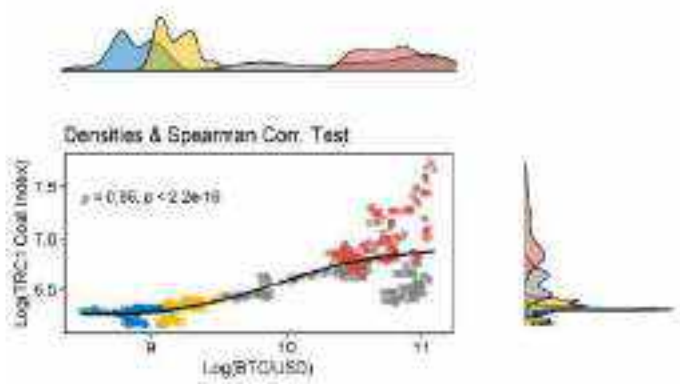
Half year correlation graph on is consistent with the previous findings between Bitcoin and Gold. During the second half year when economies felt the negative consequences most, Spearman's rho was 0.74. However, in the next period, it turned to -0.79 as a result of economic recovery around the world.



Correlation with the TRC1 Coal Index

Test	Variable	Statistic	P-value	Sample Estimate
Pearson's product-moment correlation	BTC/USD ~ TRC1 Coal Index	t = 34.854	< 2.2e-16	0.789
Kendall's rank correlation tau	BTC/USD ~ TRC1 Coal Index	z = 27.01	< 2.2e-16	0.666
Spearman's rank correlation rho	BTC/USD ~ TRC1 Coal Index	S = 9255550	< 2.2e-16	0.860

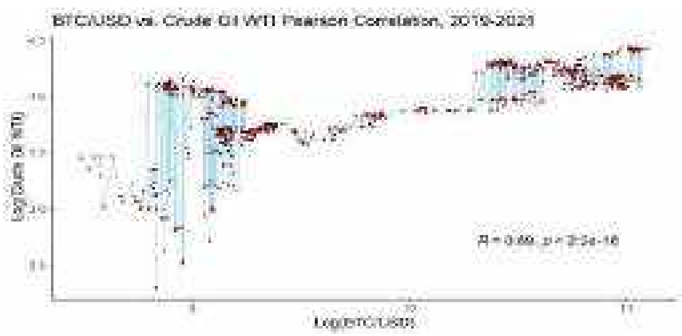


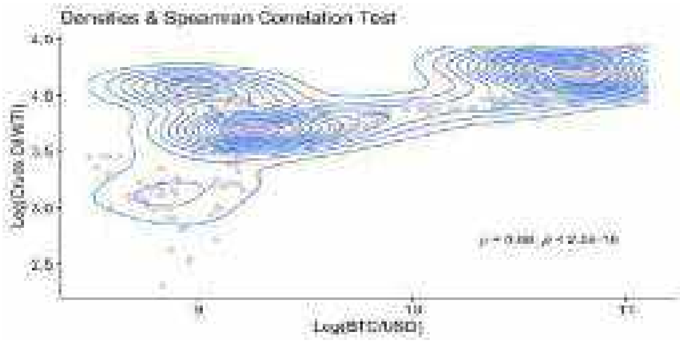


Since partial linearity is observable between BTC/USD and TRC1 Coal Index, investigating Pearson correlation can contribute to the analysis even though certain assumptions of its do not hold (normality). 0.79 Pearson's R indicates that there is a strong positive correlation between BTC/USD and TRC1 Coal Index.
With rho being 0.86, Spearman correlation test even magnifies the relationship of BTC/USD and TRC1 Coal Index compared to Pearson. Thus, the strong relationship between coal and Bitcoin is confirmed.

Correlation with the Crude Oil WTI

Test	Variable	Statistic	P-value	Sample Estimate
Pearson's product-moment correlation	BTC/USD ~ Crude Oil	t = 26.001	< 2.2e-16	0.692
Kendall's rank correlation tau	BTC/USD ~ Crude Oil	z = 18.815	< 2.2e-16	0.464
Spearman's rank correlation rho	BTC/USD ~ Crude Oil	S = 21182169	< 2.2e-16	0.678



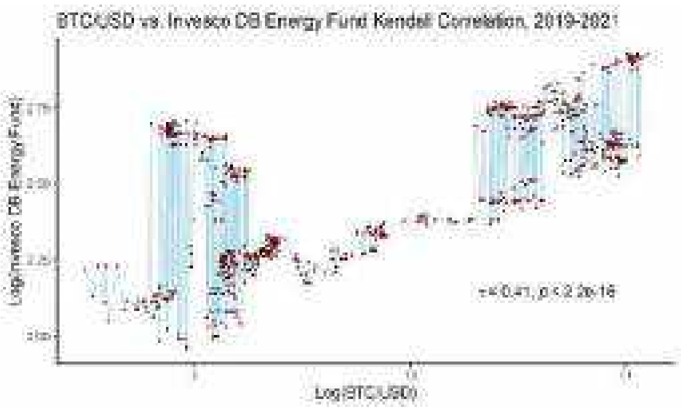


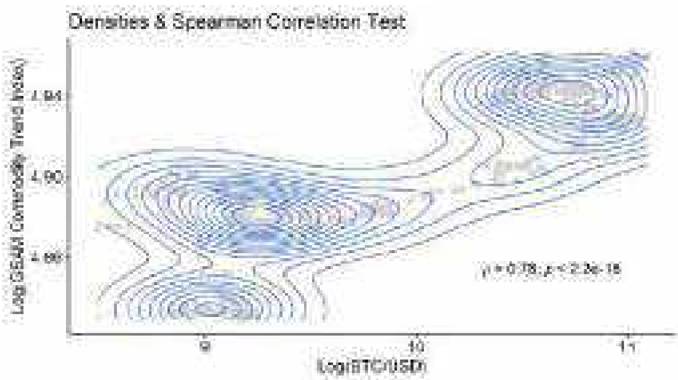
Before the tests are conducted, the outlier observation, in which Crude Oil WTI priced negatively, is omitted. Similar to BTC/USD and TRC1 Coal Index, partial linearity occurs in the BTC/USD and Crude Oil WTI relation. 0.69 Pearson's R suggests a strong positive relationship between the variables. Showing resemblance with the strength and sign of the Pearson test result, Spearman's rho calculated as 0.68.

Correlation with the Invesco DB Energy Fund & GSAM Commodity Trend Index

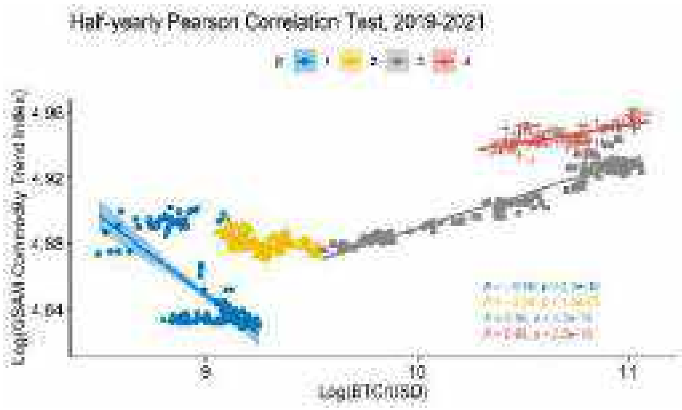
Test	Variable	Statistic	P-value	Sample
Pearson'sproduct-moment	BTC/USD ~ Invesco DB Energy Fund	t = 22.651	2.2e-16	0.641
Kendall's rank correlation tau	BTC/USD ~ Invesco DB Energy Fund	z = 16.569	2.2e-16	0.408
Spearman's rank correlation rho	BTC/USD ~Invesco DB Energy Fund	S= 787305	2.2e-16	0.549

Test	Variable	Statistic	P-value	Sample
Pearson'sproduct-moment	BTC/USD ~ GSAM Commodity Trend	t = 44.697	2.2e-16	0.855
Kendall's rank correlation tau	BTC/USD ~ GSAM Commodity Trend	z = 23.238	2.2e-16	0.573
Spearman's rank correlation rho	BTC/USD ~ GSAM Commodity Trend	S= 14299464	2.2e-16	0.783





The assumptions of linearity, normality and monotonicity fails hence, Kendall and Spearman tests should be focused. Kendall and Spearman sample estimates suggest moderate rank-based correlation (tau is equal to 0.41 & rho is 0.55). BTC/USD and the Invesco DB Energy Fund relationship can be interpreted with the other energy commodities above.
The assumption of linearity fails in addition to normality and monotonicity. Rho indicates a strong positive relationship between BTC/USD and GSAM Commodity Trend Index (0.78).

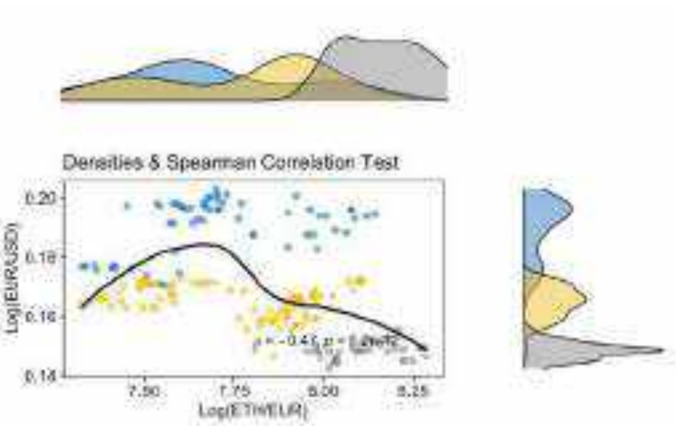


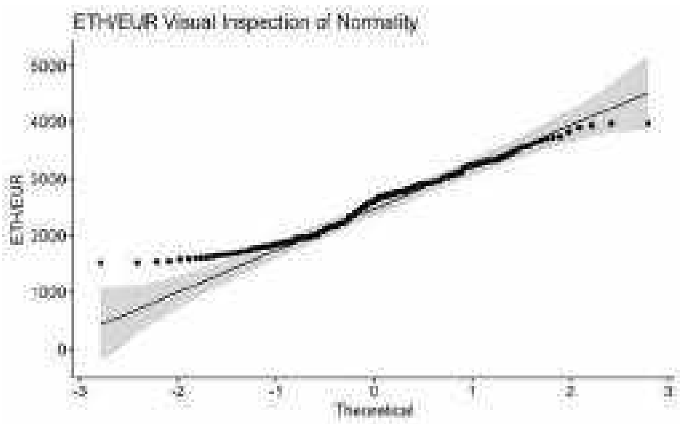
Interesting results can be attained from the decomposition of half-yearly correlations throughout the analysis horizon. The graph demonstrates that during the first half year, which includes the initial spread of COVID-19 virus throughout the countries, Pearson’s R is -0.56 and statistically significant for the relationship between BTC/USD and GSAM Commodity Trend Index. For the same variables, third half year strikes as nearly perfect correlation (0.96) indicating the significant positive signs of recovering economy.

CORRELATION ANALYSIS OVER A PANEL OF CRYPTOCURRENICES

ETH/EUR

Analysis of the Variable



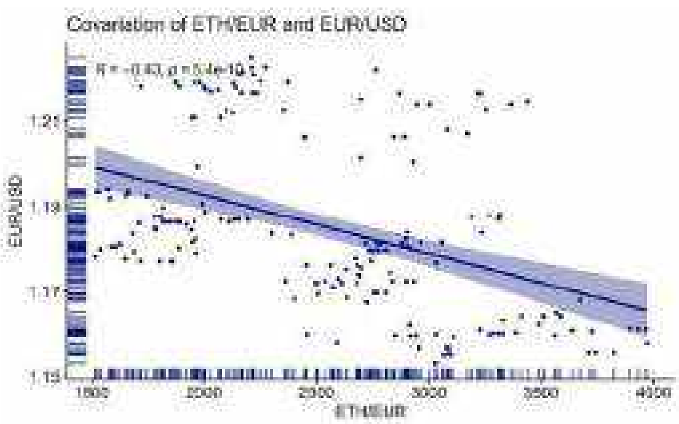


Test	Variable	Statistic	P-value
Shapiro-Wilk normality test	ETH/EUR	W = 0.96594	0.000149

The Shapiro-Wilk Test gives us a p-value of 0.000149, which is below 0.05. Therefore, we reject null hypothesis of normality of ETH/EUR, and we accept the alternative hypothesis, which is the price values of ETH/EUR are not normal. Thus, we perform a logarithmic transformation analysing returns rather than prices.

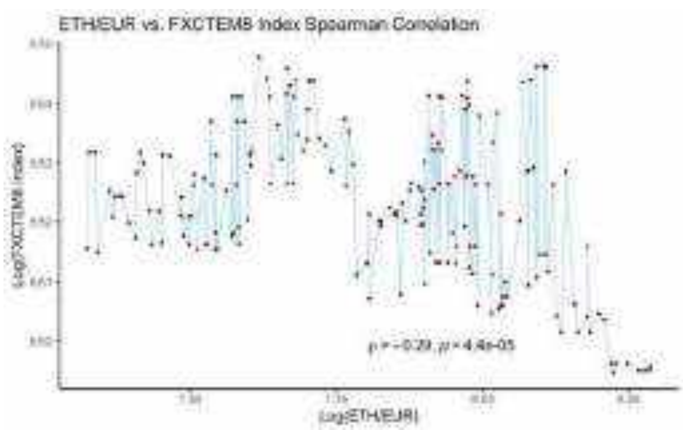
Correlation with EUR/USD

Test	Variable	Statistic	P-value	Sample Estimate
Kendall's rank correlation tau	ETH/EUR ~ EUR/USD	z = -5.5957	4.922e-08	-0.274
Spearman's rank correlation rho	ETH/EUR ~ EUR/USD	S = 610272	5.906e-10	-0.431
Pearson's product-moment correlation	ETH/EUR ~ EUR/USD	t = -5.134	7.082e-07	-0.351



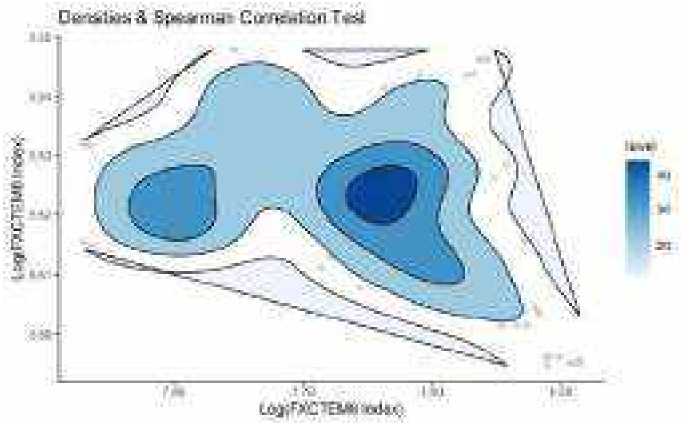
EUR/USD Is usually considered a risk off trade, performing well in low-risk environment. Thus, these results are consistent with a view where ETH/EUR Is a risky trade.

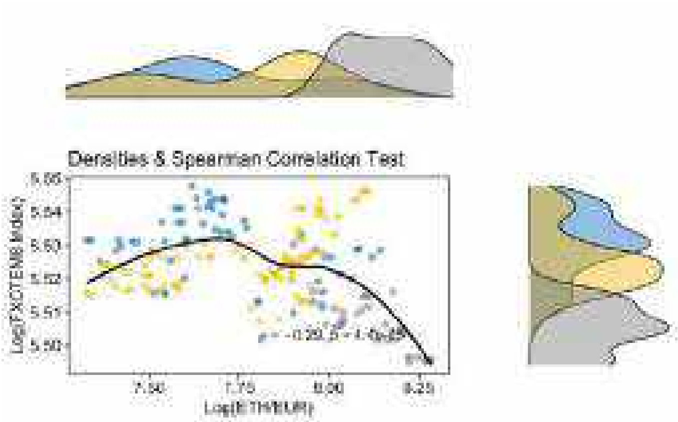
Correlation with FXCTEM8 Index



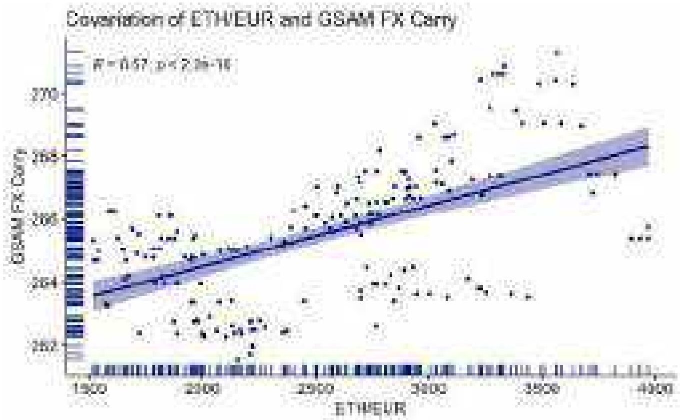
Test	Variable	Statistic	P-value	Sample Estimate
Kendall's rank correlation tau	ETH/EUR ~ FXCTEM8 Index	z = -3.9933	6.515e-05	-0.195
Spearman's rank correlation rho	ETH/EUR ~ FXCTEM8 Index	S =454350	4.409e-05	-0.292
Pearson's product-moment correlation	ETH/EUR ~ FXCTEM8 Index	t = -4.9276	1.827e-06	-0.339

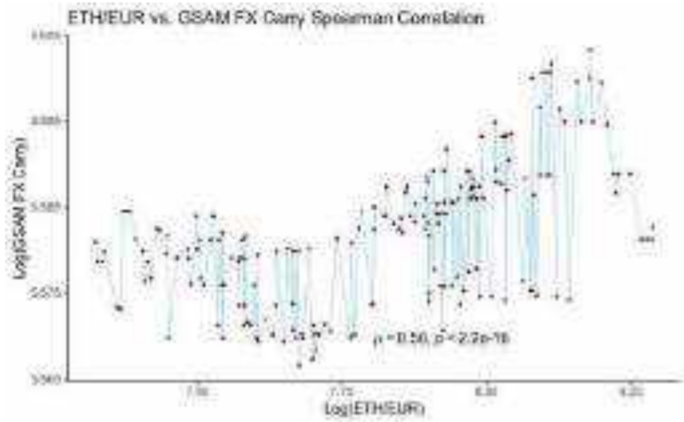
We can see the crypto's weak but negative correlation with 8 emerging market's currencies. The reason is simple: ETH's value increases against a G10 currency and as the index's currencies depreciate between period of 1st of May and 5th of November. Especially strong decrease in value of Emerging Market currencies after first week of September and bullish pattern of ETH strengthened the negative correlation. As it can be seen above very clearly.





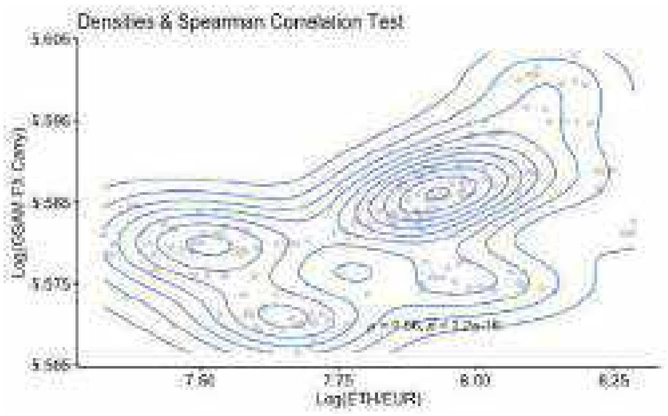
Correlation with GSAM FX Carry



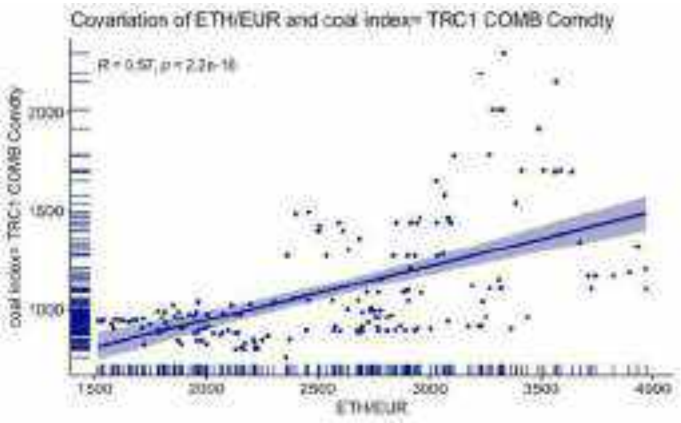


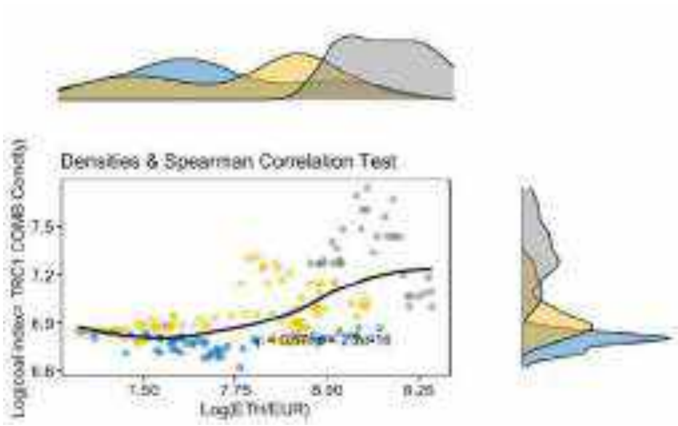
Test	Variable	Statistic	P-value	Sample Estimate
Kendall's rank correlation tau	ETH/EUR ~ GSAM FX Carry	z = 8.0857	6.181e-16	0.396
Spearman's rank correlation rho	ETH/EUR ~ GSAM FX Carry	S = 496599	< 2.2e-16	0.558
Pearson's product-moment correlation	ETH/EUR ~ GSAM FX Carry	t = 8.8275	7.606e-16	0.542

Unlike two previous currency pairs we realize that GSAMFX Carry has moderate strength positive correlation. The reason is embedded in the Index's nature. Since the Index invests in highest yielding currencies and shorts lowest yielding ones and the Index value has gained value recently.



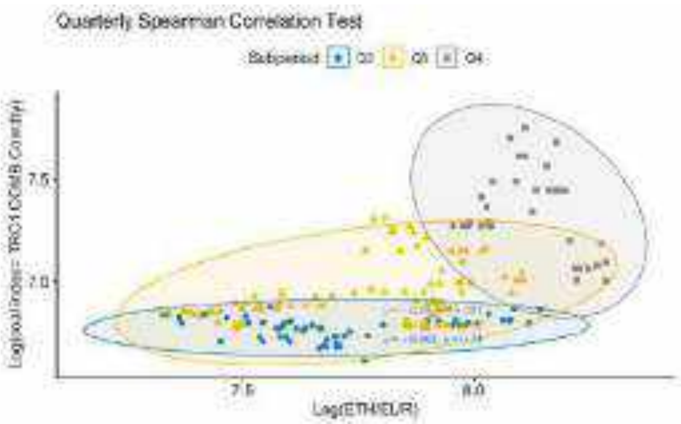
Correlation with TRC1 COMB Comdty

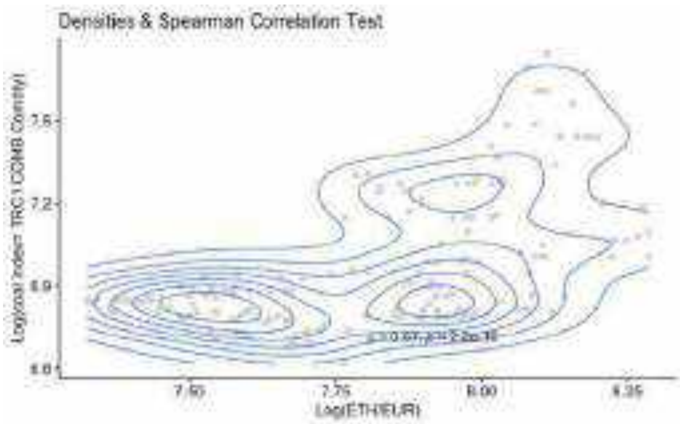




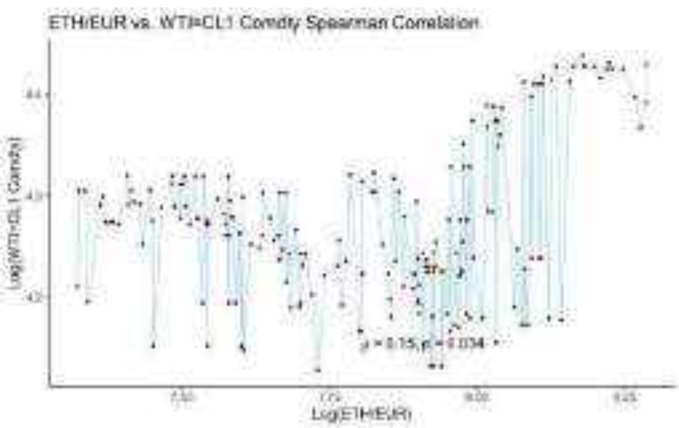
Test	Variable	Statistic	P-value	Sample Estimate
Kendall's rank correlation tau	ETH/EUR ~ TRC1 COMB Comdty	z = 7.3891	1.478e-3	0.362
Spearman's rank correlation rho	ETH/EUR ~ TRC1 COMB Comdty	S= 524616	2.609e-5	0.533
Pearson'sproduct-moment correlation	ETH/EUR ~ TRC1 COMB Comdty	t = 8.7253	1.447e-5	0.537

ETH/EUR returns are positively correlated with TRC1 commodity index. The logarithmic transformation value of TRC1 Commodities has increased from around 6.6 to almost 8 in Q4. And as we can see from the densities of ETH/EUR, the crypto's returns are concentrated in high values greater than 7.75 as response to increase in commodity prices. Thus, a positive and significant relationship between coal and ETH is uncovered.





Correlation with CL1 Comdty (WTI)

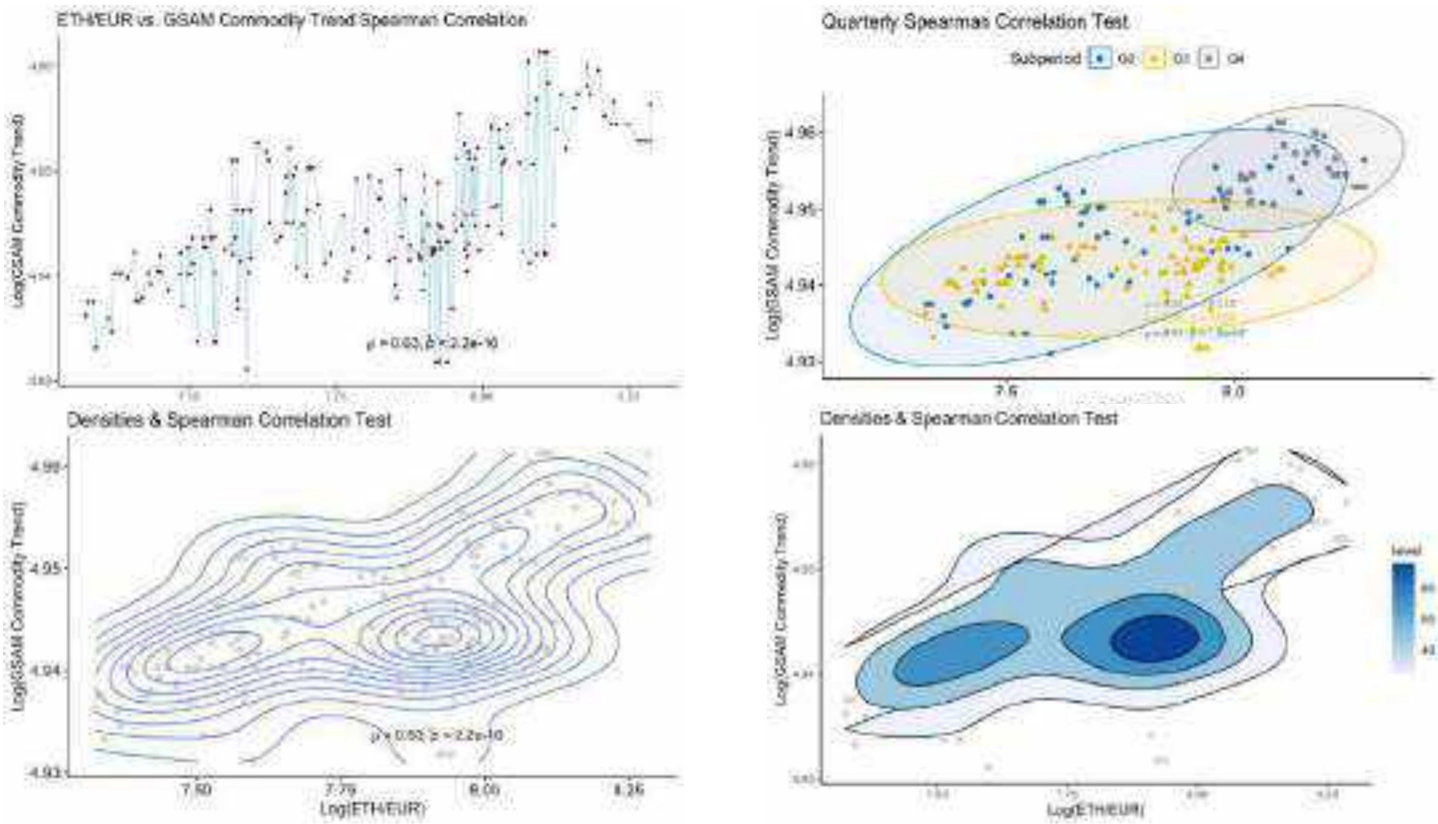


Test	Variable	Statistic	P-value	Sample Estimate
Kendall's rank correlation tau	ETH/EUR ~ WTI=CL1 Comdty	z = 1.0685	0.2853	0.052
Spearman's rank correlation rho	ETH/EUR ~ WTI=CL1 Comdty	S = 992798	0.1069	0.117
Pearson's product-moment correlation	ETH/EUR ~ WTI=CL1 Comdty	t = 3.2602	0.001323	0.231

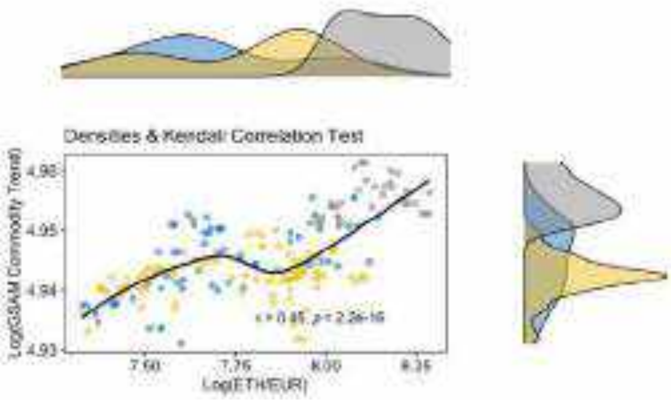
There is weak correlation between returns of these two assets. However, in Q4 strong positive correlation with ETH is due to WTI increase in value. Overall, a decreasing trend in Q3 caused a lower correlation strength.

Correlation with GSAM Commodity

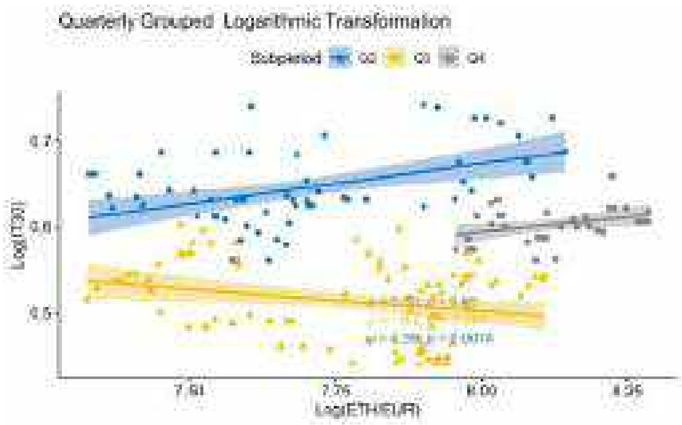
Test	Variable	Statistic	P-value	Sample
Kendall's rank correlation tau	ETH/EUR ~ GSAM Commodity	z=9.1288	< 2.2e-16	0.447
Spearman's rank correlation rho	ETH/EUR ~ GSAM Commodity	S= 18682	< 2.2e-16	0.627
Pearson'sproduct-moment	ETH/EUR ~ GSAM Commodity	t= 0.651	< 2.2e-16	0.614



As we can see from Quarterly Spearman Correlation graph above all of the quarters suggest the positive correlation, meaning ETH/EUR has shown a consistent behaviour with GSAM Commodity Trend an index based on trending commodities. Therefore, ETH show stronger correlation with a basket of commodities rather than a single commodity, such as oil.

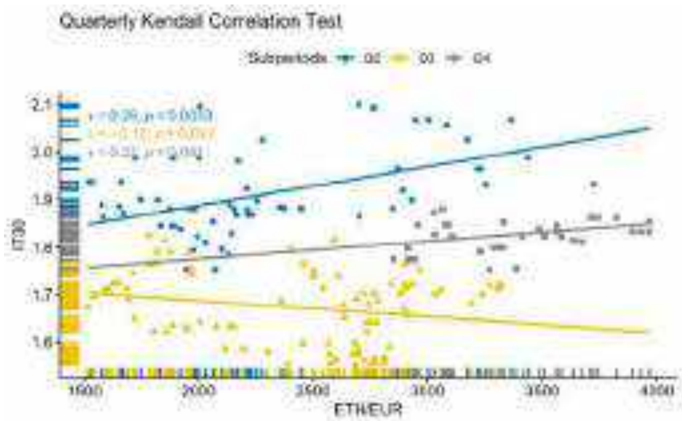


Correlation with IT30



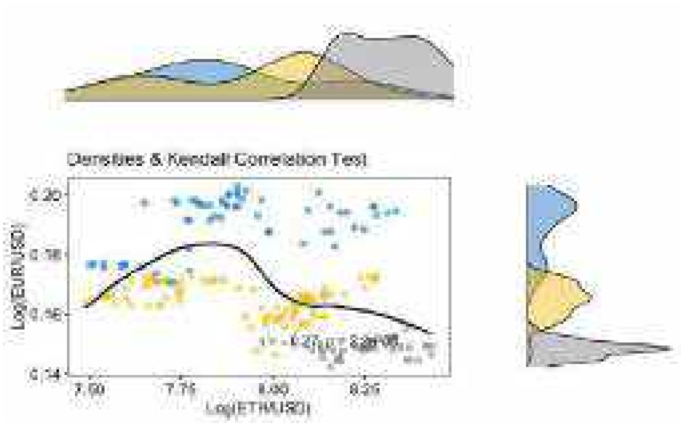
Test	Variable	Statistic	P-value	Sample Estimate
Kendall's rank correlation tau	ETH/EUR ~ IT30	z = 1.0628	0.2879	0.052
Spearman's rank correlation rho	ETH/EUR ~ IT30	S = 1046682	0.3401	0.069
Pearson's product-moment correlation	ETH/EUR ~ IT30	t = 0.58239	0.561	0.042

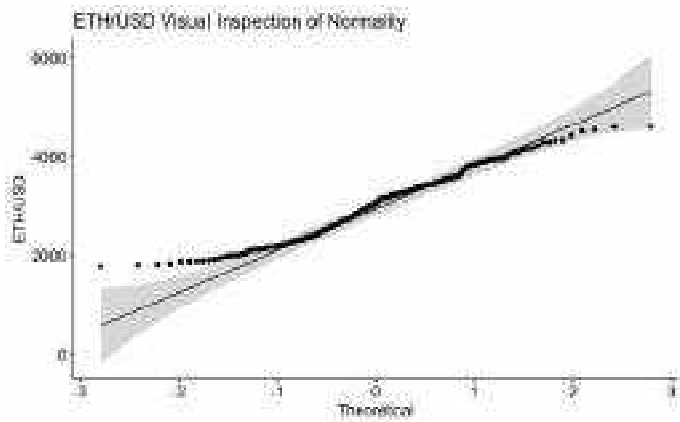
As we can see, the Italian 30-year government bonds have the weakest correlation with ETH. Government bonds are low risk investments. Since, crypto investors can be considered to be more risk taker, we can assume that weak correlation is a result of different investor risk appetite for the two different assets. This may also support the idea of ETH as a completely decentralized asset, not being affected by inflation, interest rate increase or central bank macroeconomic policies.



ETH/USD

Analysis of the Variable



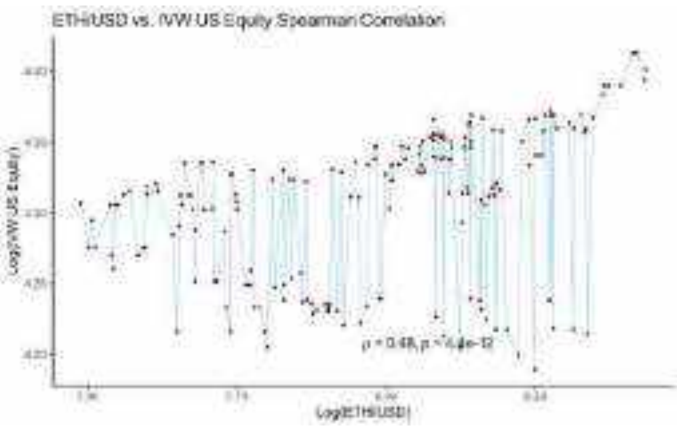


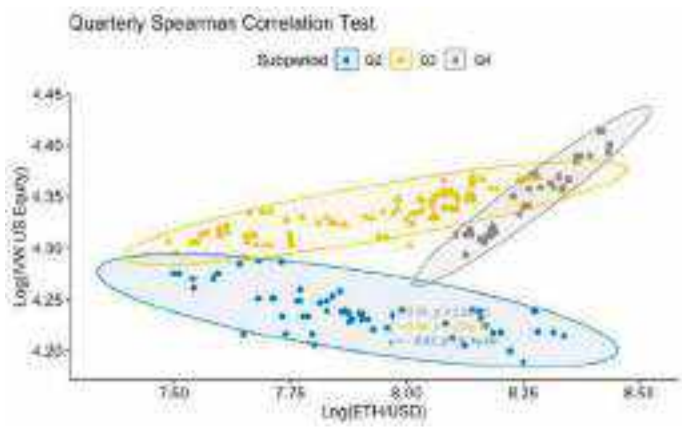
Test	Variable	Statistic	P-value
Shapiro-Wilk normality test	ETH/USD	W = 0.97022	0.0004684

The Shapiro-Wilk Test gives us a p-value of 0.0004684, which is below 0.05. Therefore, we reject null hypothesis of normality of ETH/USD, and we accept the alternative hypothesis, which is the price values of ETH/USD are not normal. Thus, we will model this variable according to its returns.

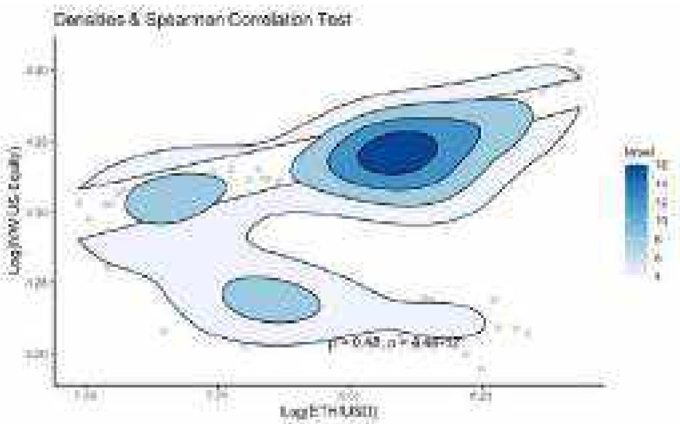
Correlation with IVW US Equity

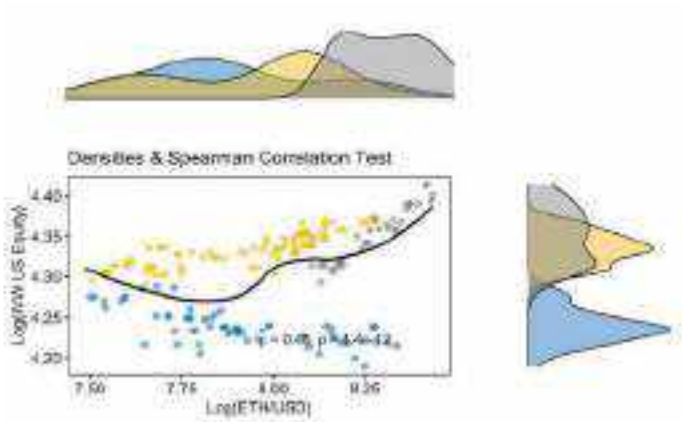
Test	Variable	Statistic	P-value	Sample Estimate
Kendall's rank correlation tau	ETH/USD ~ IVW US Equity	z = 6.9677	3.221e-12	0.341
Spearman's rank correlation rho	ETH/USD ~ IVW US Equity	S = 589485	4.389e-12	0.476
Pearson's product-moment correlation	ETH/USD ~ IVW US Equity	t = 5.7304	3.951e-08	0.386





ETH and IVW equity index are similarly volatile and risky trades. Therefore, both assets tend to attract high risk taker investors. In addition, Ethereum's bullish season in Q3 and especially in Q4 can be clearly seen in Quarterly Spearman Correlation graphs as rho hits 0.86 and 0.95 respectively.

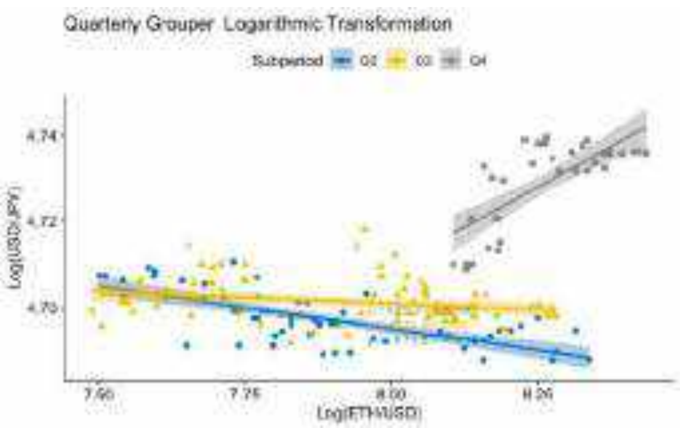


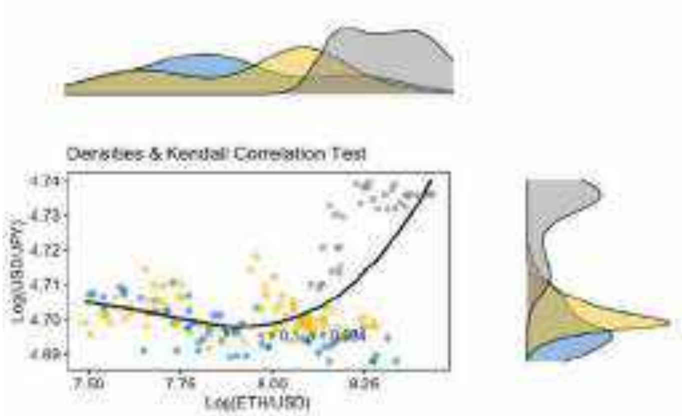


Correlation with USD/JPY

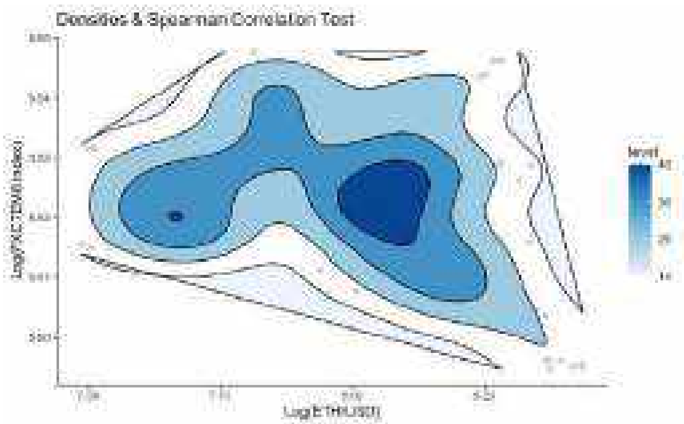
Test	Variable	Statistic	P-value	Sample Estimate
Kendall's rank correlation tau	ETH/USD ~ USD/JPY	z = 2.1222	0.03382	0.104
Spearman's rank correlation rho	ETH/USD ~ USD/JPY	S = 935827	0.02063	0.168
Pearson'sproduct-moment correlation	ETH/USD ~ USD/JPY	t = 6.088	6.348e-09	0.406

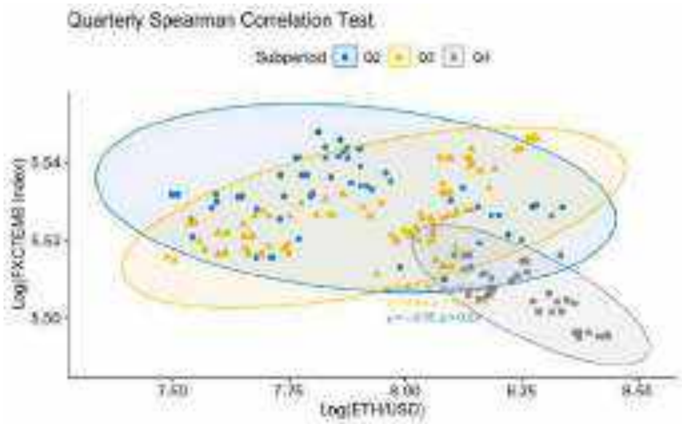
The correlation between ETH/USD and USD/JPY is surprisingly positive. However, with a closer inspection the relationship is typically negative but in the last few months. JPY has in fact appreciated versus the USD.

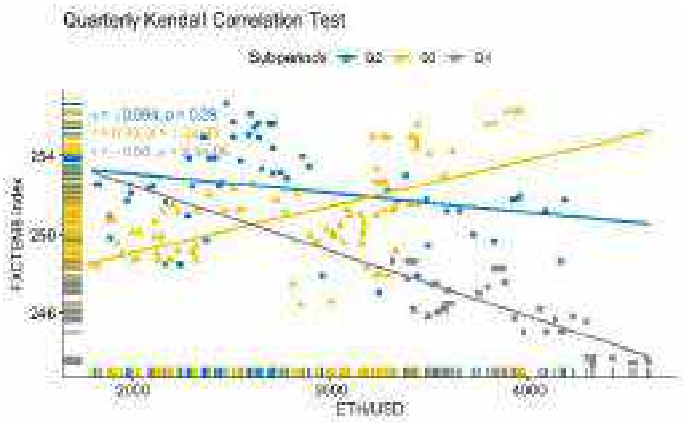




Correlation with FXCTEM8 Index





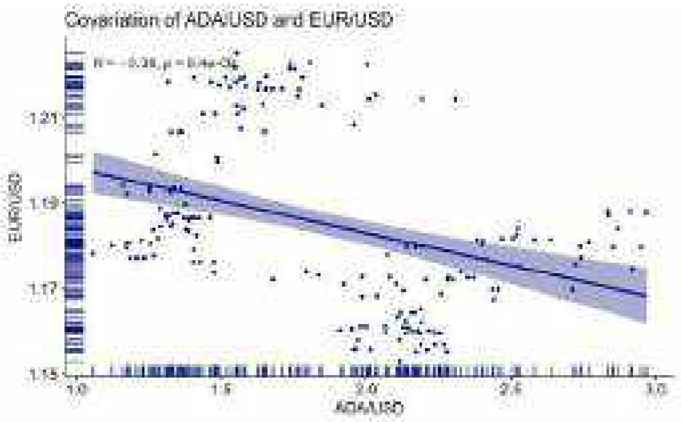


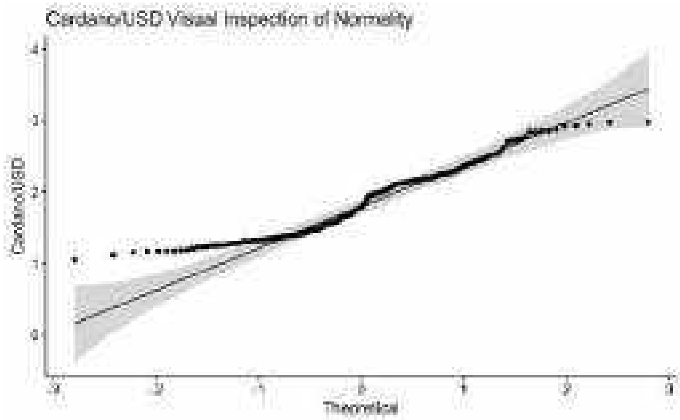
Test	Variable	Statistic	P-value	Sample
Kendall's rank correlation tau	ETH/USD ~ FXCTEM8 Index	z = -3.6667	0.0002457	-0.179
Spearman's rank correlation rho	ETH/USD ~ FXCTEM8 Index	S = 1427145	0.0001887	-0.268
Pearson'sproduct-moment	ETH/USD ~ FXCTEM8 Index	t = -4.3166	2.566e-05	-0.301

ETH/USD has a significant negative correlation with emerging market currencies indexed according to the FXCTEM8 index. This comes as no surprise given the performance of EM currencies in the last few months.

ADA/USD

Analysis of the Variable

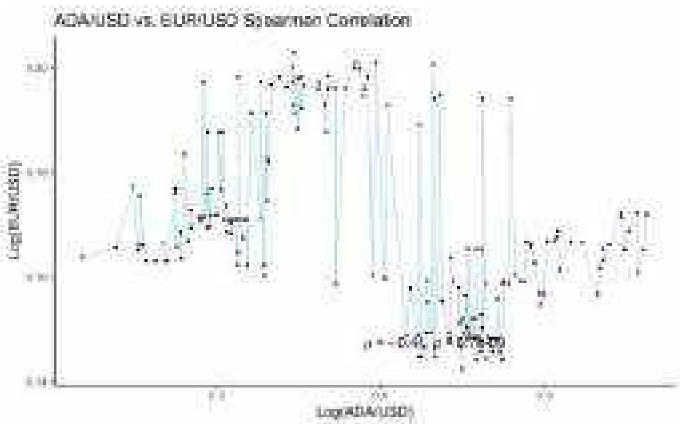


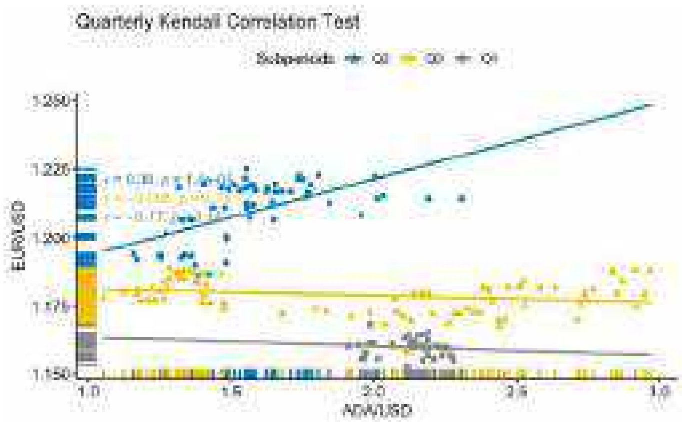


Test	Variable	Statistic	P-value
Shapiro-Wilk normality test	ADA/USD	W =0.94058	4.957e-07

Since p-value is lower than 0.05, we can say that ADA/USD is not normal and not linear. As it is visible in graphs. Thus, we model returns.

Correlations with EUR/USD



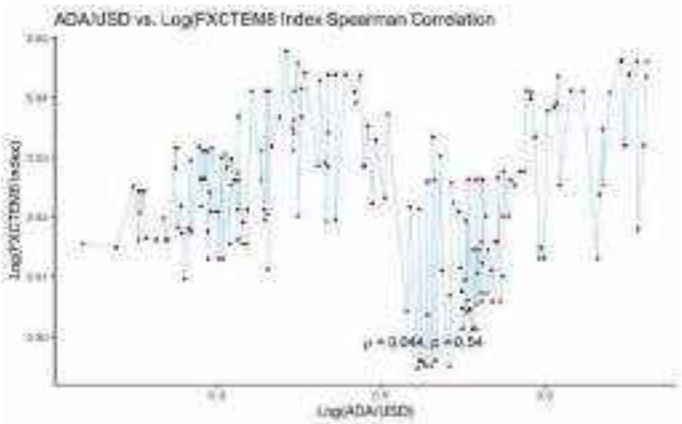


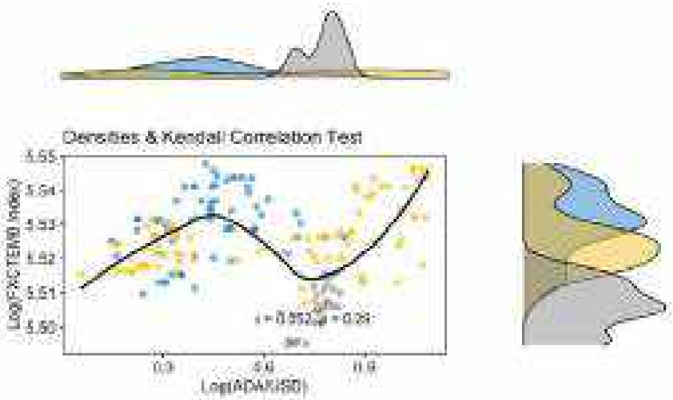
Test	Variable	Statistic	P-value	Sample
Kendall's rank correlation tau	ADA/USD ~ EUR/USD	z = -4.013	5.994e-05	-0.197
Spearman's rank correlation rho	ADA/USD ~ EUR/USD	S = 1582214	6.668e-09	-0.406
Pearson's product-moment	ADA/USD ~ EUR/USD	t = -5.5057	1.202e-07	-0.373

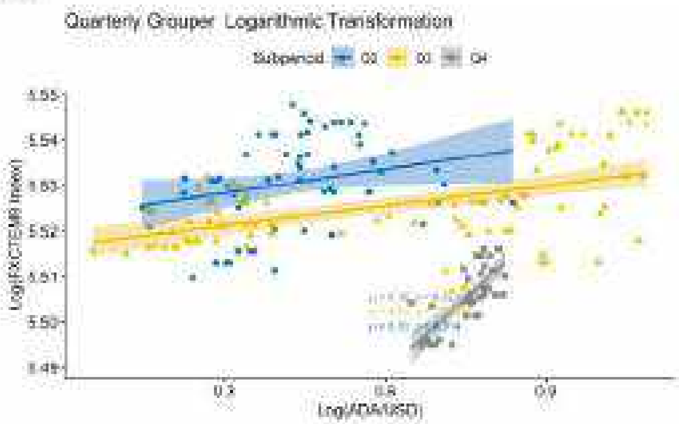
ADA/EUR shows a negative relationship with EUR/USD, this is consistent with the recent strengthening of the dollar. It is however noticeable a positive relationship throughout Q2.

Test	Variable	Statistic	P-value	Sample
Kendall's rank correlation tau	ADA/USD ~ FXCTEM8 Index	z = 1.0673	0.2858	0.052
Spearman's rank correlation rho	ADA/USD ~ FXCTEM8 Index	S = 1075196	0.5439	0.009
Pearson's product-moment	ADA/USD ~ FXCTEM8 Index	t = 0.13283	0.8945	0.009

Correlations with FXCTEM8 Index



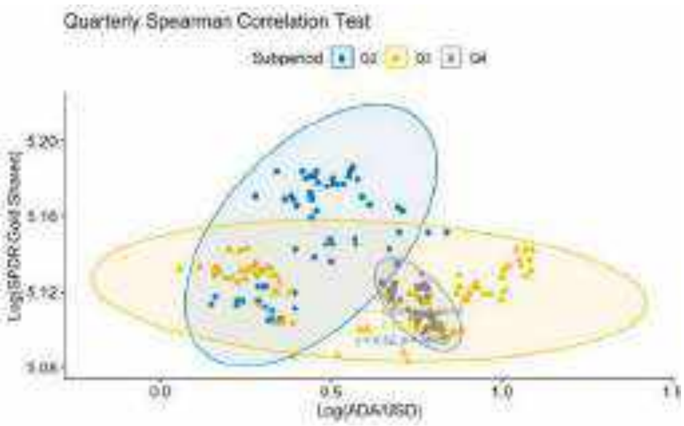


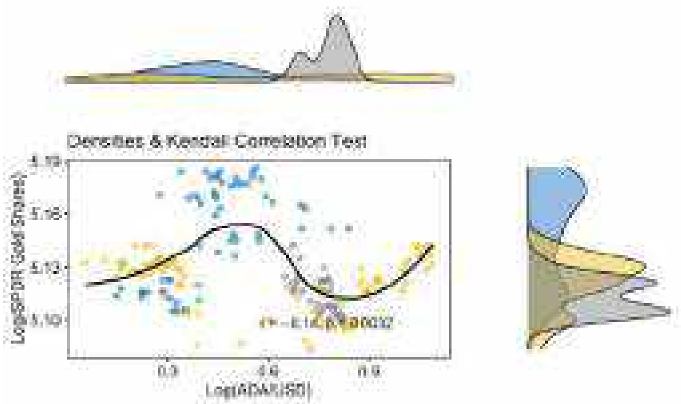


Unlike Ethereum's correlation with FXCTEM8 Index's ADA/USD returns are positively but weakly correlated with the Index. The most obvious reason is ADA's depreciation in Q4 against USD and the index's similar behaviour. As it can be seen in Q4 correlation's rho value of 0.76.

Correlations with SPDR Gold Share

Test	Variable	Statistic	P-value	Sample Estimate
Kendall's rank correlation tau	ADA/USD ~ SPDR Gold Share	z = -2.9525	0.003152	-0.144
Spearman's rank correlation rho	ADA/USD ~ SPDR Gold Share	S = 1383606	0.001477	-0.229
Pearson's product-moment correlation	ADA/USD ~ SPDR Gold Share	t = -2.9896	0.003169	-0.213

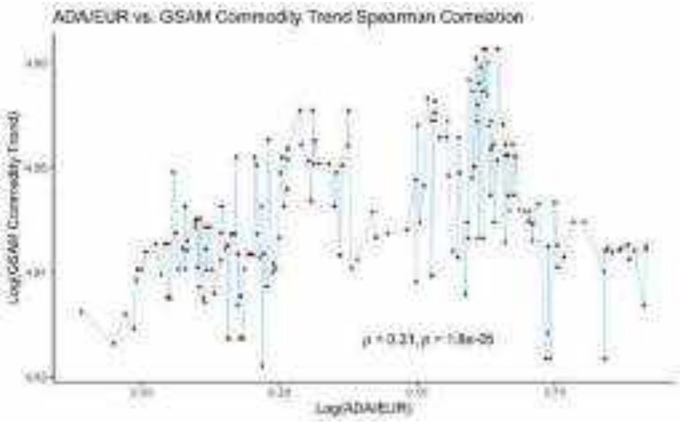


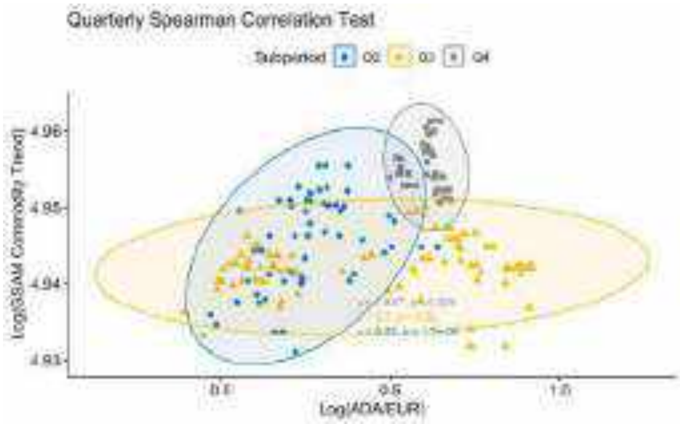


We realize a rather strong negative correlation between gold and ADA in 4th quarter. This is due to the decline in ADA after October 1st while Gold Share has increased. As we can see from the density's graphs in Q4 there is a concentration of lower price for ADA compared to the Gold Share. Nevertheless, overall, we can see the negative correlation between the crypto and gold. The narrative that compares cryptos to gold is thus questionable and should be carefully checked.

ADA/EUR-Correlations with GSAM Commodity Trend

Test	Variable	Statistic	P-value	Sample
Kendall's rank correlation tau	ADA/EUR ~ GSAM Commodity	z = 4.1854	2.846e-05	0.205
Spearman's rank correlation rho	ADA/EUR ~ GSAM Commodity	S = 780419	1.803e-05	0.306
Pearson'sproduct-moment	ADA/EUR ~ GSAM Commodity	t = 4.1072	5.982e-05	0.287

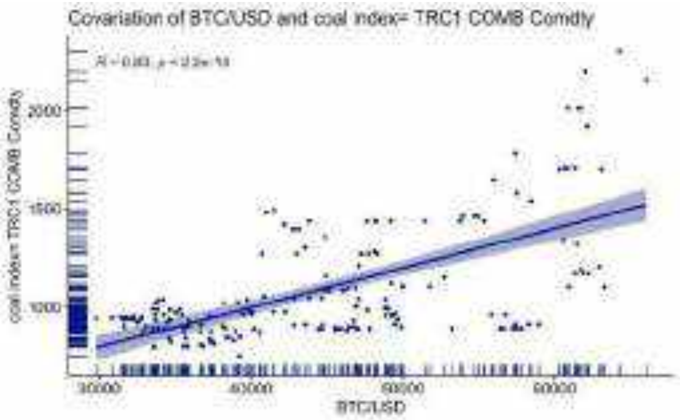


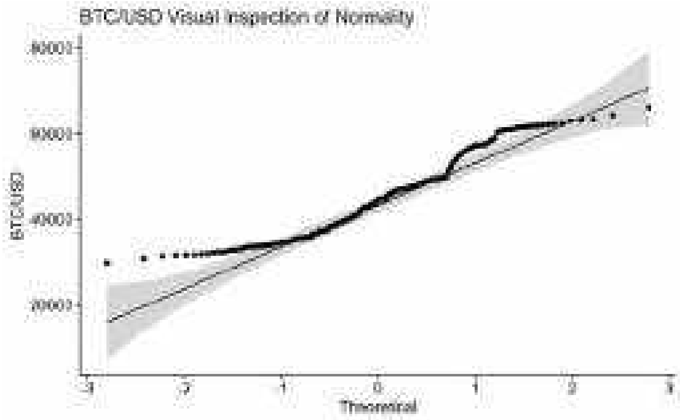


Positive correlation between ADA/EUR and GSAM Commodity Index comes as no surprise. Commodities appear to be significantly correlated with cryptos, especially when analysed through indices rather than futures.

BTC/USD

Analysis of the Variable





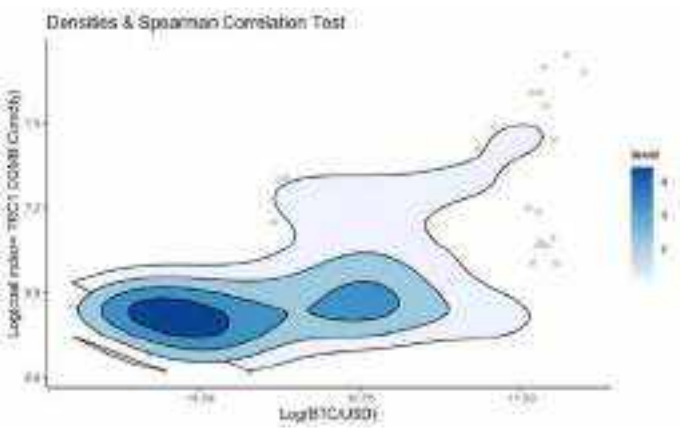
Test	Variable	Statistic	P-value
Shapiro-Wilk normality test	BTC/USD	W =0.94276	7.633e-07

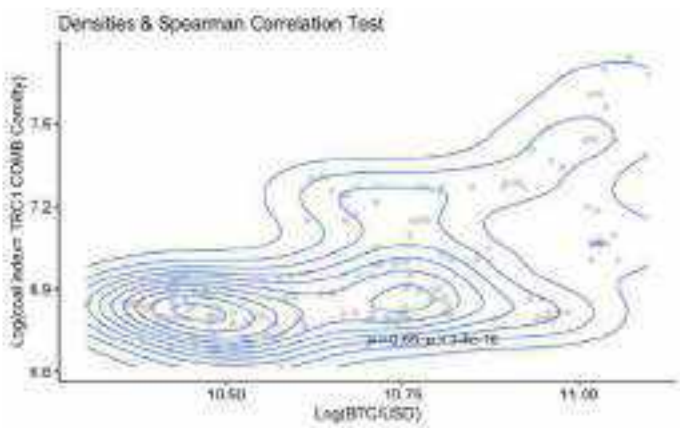
Since BTC's p-value is lower than 0.05 we need to reject null hypothesis that BTC/USD prices are normal. Thus, we model returns.

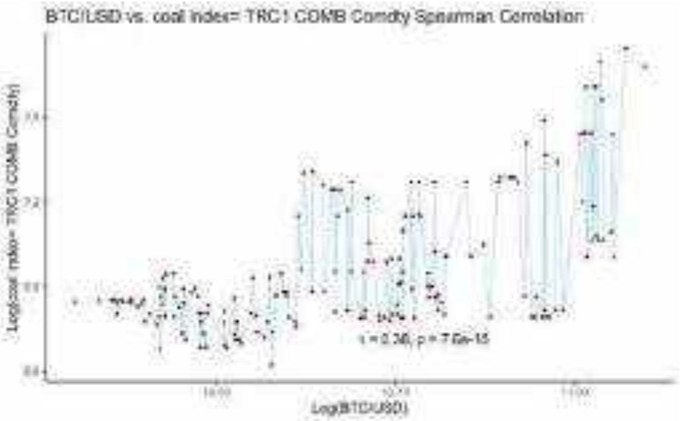
Correlations with TRC1 COMB Comdty

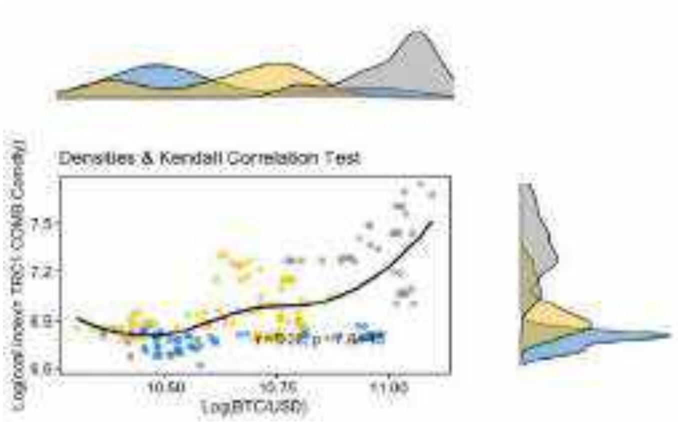
Test	Variable	Statistic	P-value	Sample
Kendall's rank correlation tau	BTC/USD ~ TRC1 COMB Comdty	z = 7.7733	7.647e-5	0.381
Spearman's rank correlation rho	BTC/USD ~ TRC1 COMB Comdty	S = 508696	3.365e-6	0.547
Pearson'sproduct-moment	BTC/USD ~ TRC1 COMB Comdty	t = 10.685	2.2e-16	0.615

Bitcoin's strong positive correlation with coal index proves how important energy prices are for Bitcoin. Bitcoin mining requires significant amount of energy consumption thus energy cost may be priced in the BTC. As coal prices increase, we see ever more increasing Bitcoin prices.



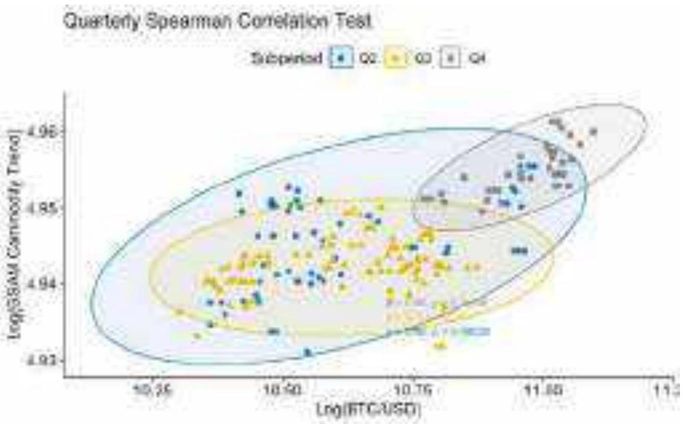


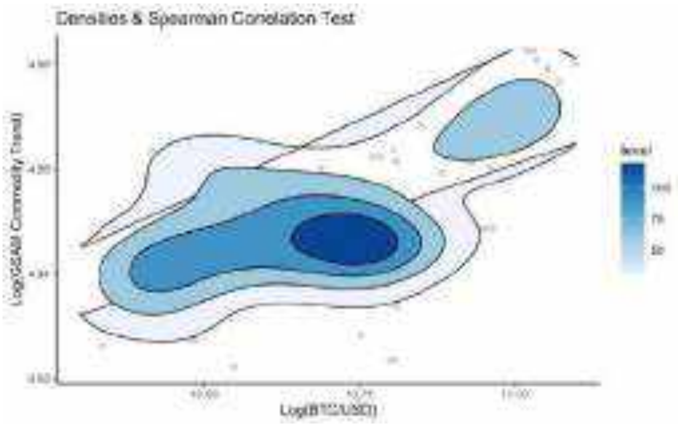


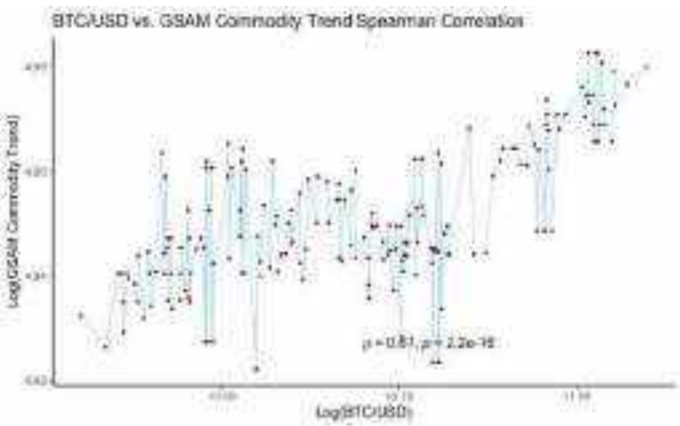


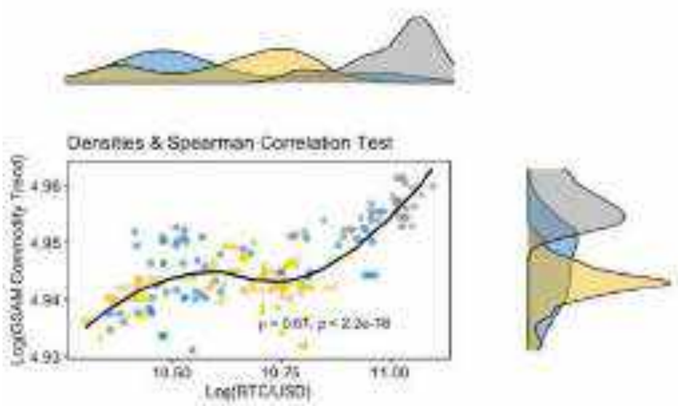
Correlations With GSAM Commodity Trend

Test	Variable	Statistic	P-value	Sample
Kendall's rank correlation tau	BTC/USD ~ GSAM Commodity	z = 8.954	< 2.2e-16	0.439
Spearman's rank correlation rho	BTC/USD ~ GSAM Commodity	S = 439185	< 2.2e-16	0.609
Pearson'sproduct-moment	BTC/USD ~ GSAM Commodity	t = 11.541	< 2.2e-16	0.644





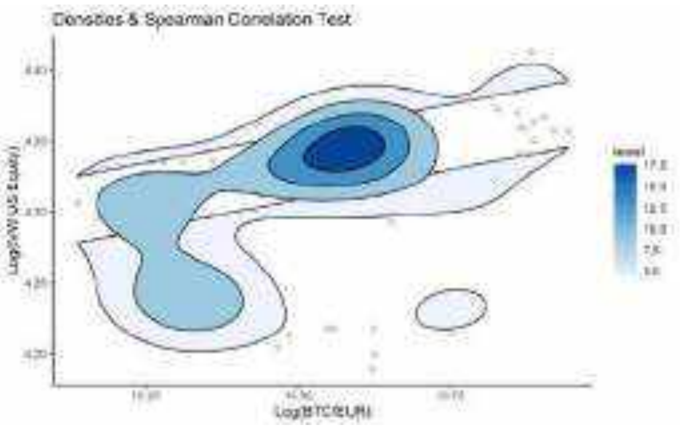


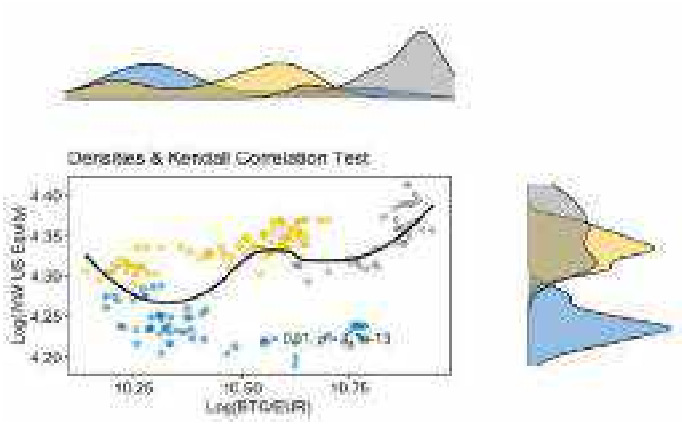


As we have discovered in Ethereum, cryptocurrencies tend to have a stronger positive relation with indexes which include various types of commodities. This proves cryptocurrencies' exposure to volatile commodity prices. For example, Q4 has delivered positive returns for both BTC and Commodity trend, as we can see from the density and Spearman chart. In addition, it is seen a persistent coherence between BTC and the Commodity trend in Q2, Q3, Q4, when correlations are always positive.

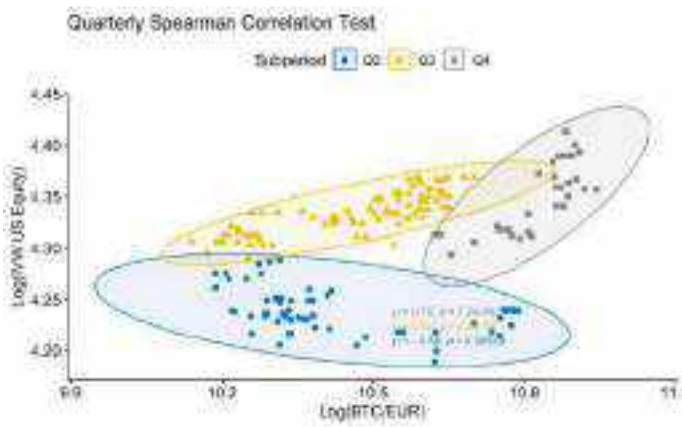
BTC/EUR-Correlations with IVW US Equity

Test	Variable	Statistic	P-value	Sample
Kendall's rank correlation tau	BTC/USD ~ IVW US Equity	z = 7.1863	6.658e-13	0.352
Spearman's rank correlation rho	BTC/USD ~ IVW US Equity	S = 555721	1.094e-13	0.506
Pearson'sproduct-moment	BTC/USD ~ IVW US Equity	t = 6.9037	7.656e-11	0.450



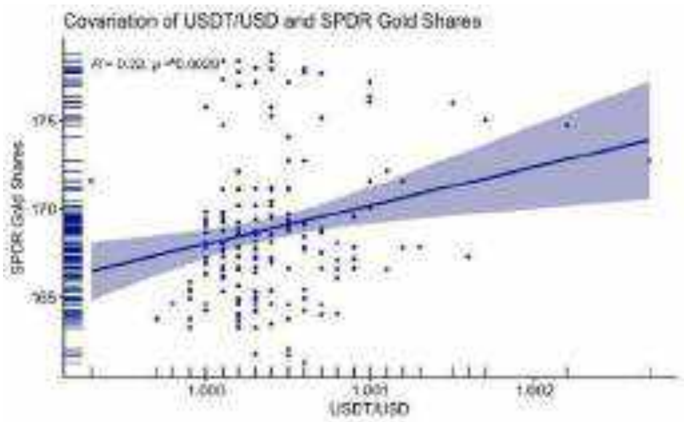


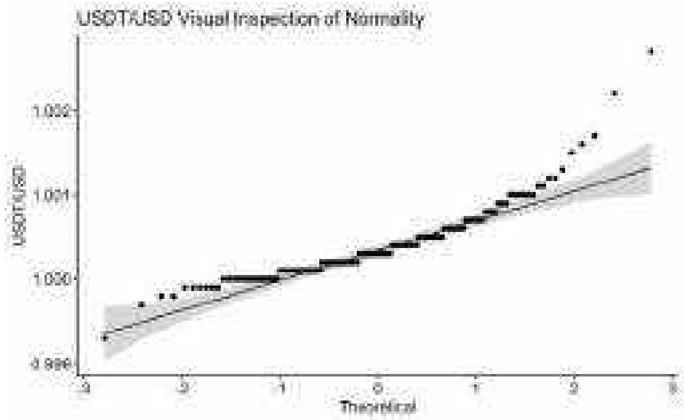
Just like Ethereum we observe very strong positive correlations. Both US growth stocks and BTC/EUR performed well in third and especially in fourth quarter we can observe really strong positive correlations. Judging from the density graphs concentration in high returns especially in Equity index has increased the coefficient of correlation.



USDT/USD

Analysis of the Variable



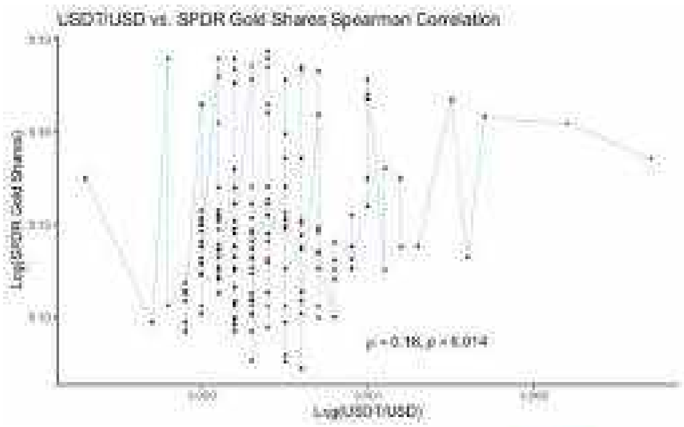


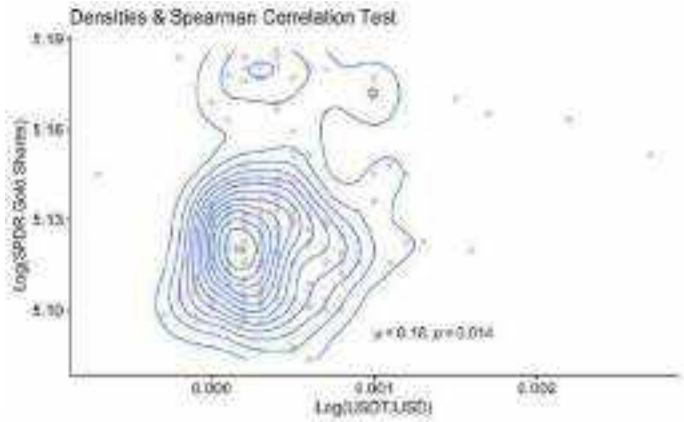
Test	Variable	Statistic	P-value
Shapiro-Wilk normality test	USDT/USD	W = 0.86638	9.381e-12

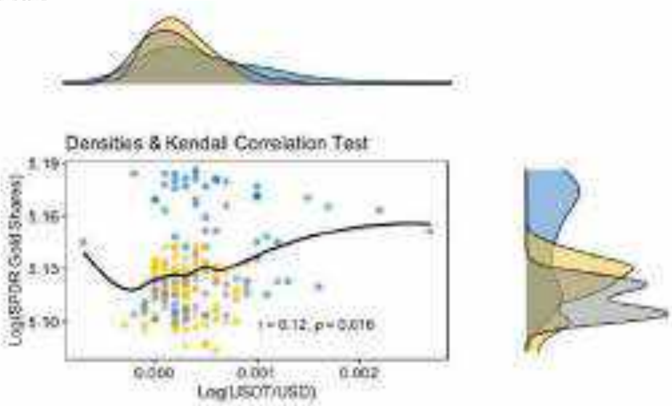
Since USDT's p-value is lower than 0.05 we reject the null hypothesis and accept that USDT prices are not normal. Thus, we model its returns.

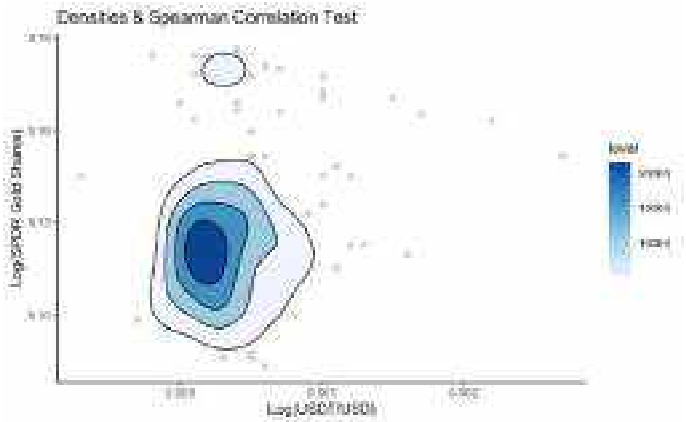
Correlations With SPDR Gold Shares

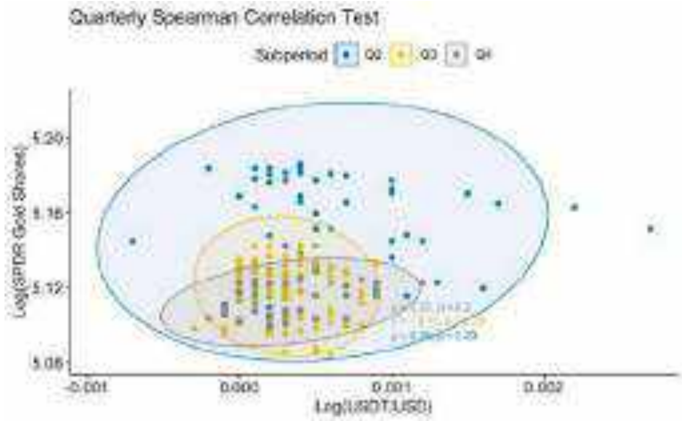
Test	Variable	Statistic	P-value	Sample Estimate
Kendall's rank correlation tau	USDT/USD ~ SPDR Gold Shares	z = 2.4166	0.01567	0.124
Spearman's rank correlation rho	USDT/USD ~ SPDR Gold Shares	S = 879265	0.01388	0.180
Pearson's product-moment correlation	USDT/USD ~ SPDR Gold Shares	t = 3.0263	0.00283	0.217







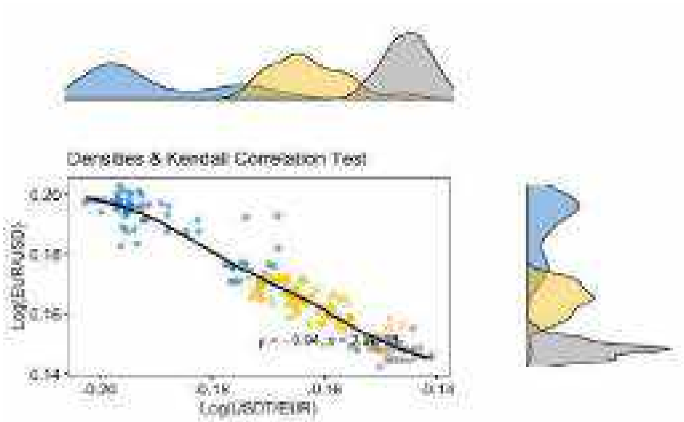


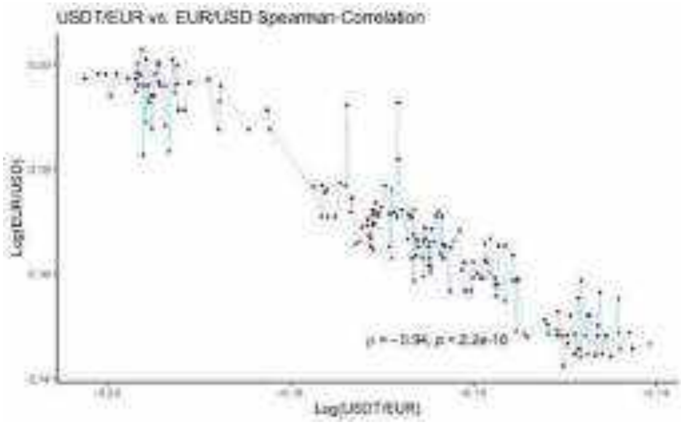


USDT is pegged to USD and is claimed by Tether to have intrinsic value, meaning every USDT is backed either a precious metal or fiat currency. Therefore, it shows significantly different dynamics compared to other three cryptos analysed so far. For example, USDT/USD shows extremely weak positive correlation with GSAM commodity index while the other three cryptos show a positive correlation between 0.4 and 0.6. Even DB Invesco Energy Fund correlation with USDT/USD is weakly negatively correlated.

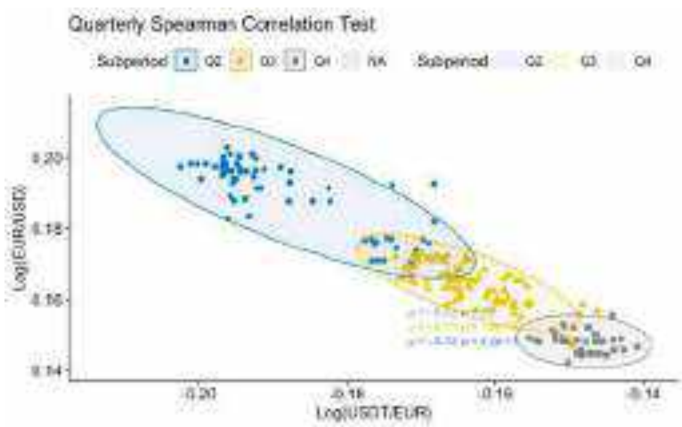
Correlations with EUR/USD

Test	Variable	Statistic	P-value	Sample Estimate
Kendall's rank correlation tau	USDT/USD ~ EUR/USD	z = -15.696	< 2.2e-16	-0.777
Spearman's rank correlation rho	USDT/USD ~ EUR/USD	S = 2077261	< 2.2e-16	-0.936
Pearson's product-moment correlation	USDT/USD ~ EUR/USD	t =-44.011	< 2.2e-16	-0.955



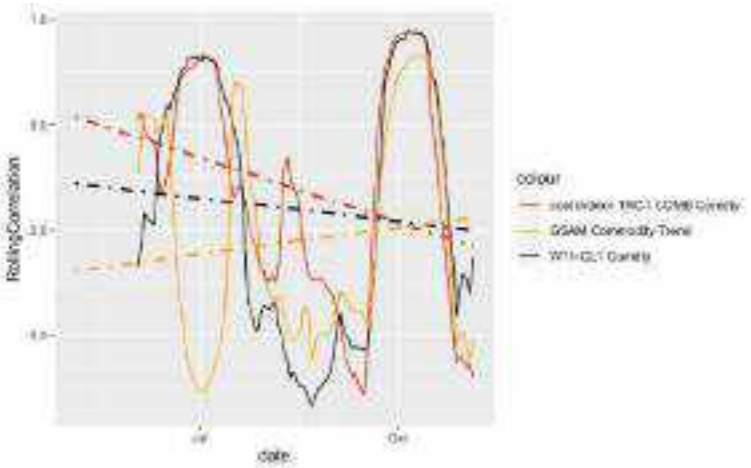


USDT has a behaviour consistent with EUR/USD, since Euro depreciated against USD since May, we see an almost perfect and extremely strong negative correlation between USDT/EUR and EUR/USD as expected. Investment in USDT almost bears the same risk of investing in USD. Thus, the market seems to believe in USDT replication of USD dollar.



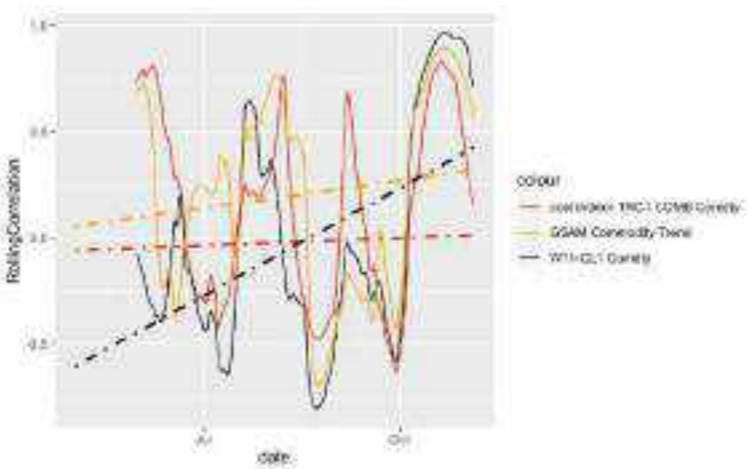
ROLLING CORRELATIONS

Commodities Rolling Correlation with USDT/EUR



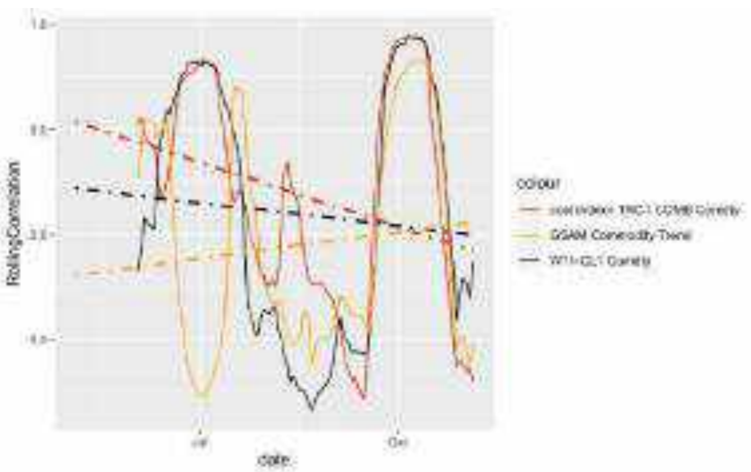
Rolling analysis through the recursive computation of correlations allows us to highlight patterns in correlation dynamics. In this first chart USDT/EUR is analysed. We can see that WTI and TRC1 Comdty have a similar behaviour in respect to USDT/EUR. Interestingly GSAM commodity trend is negatively correlated with USDT/EUR and behaves differently throughout Q2. The correlation of these commodities indices after a peak in October has turned now negative.

Commodities Rolling Correlation with BTC/EUR



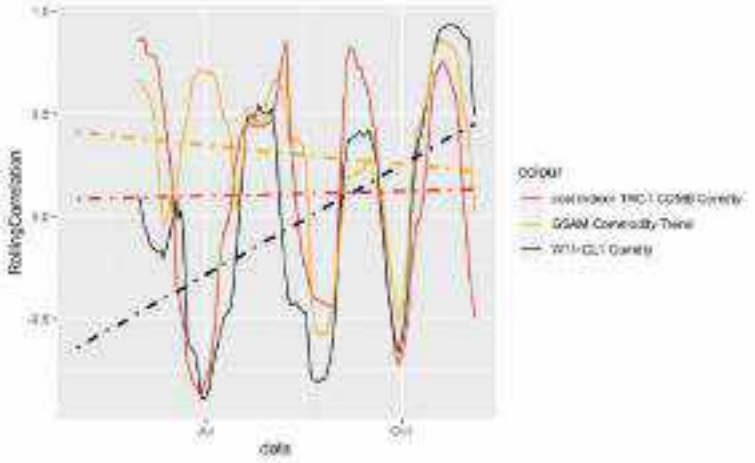
BTC/EUR shows fewer clear patterns. It can be assessed also from this charts that correlations of BTC with coal, WTI and GSAM commodity trend is quite volatile. It can be seen that after a peak in October correlation is decreasing. Thus, if we believe that commodities prices are priced in BTC a bearish view could be sustained.

Commodities Rolling Correlation with ADA/EUR



ADA/EUR shows a significantly different behaviour in respect to commodities. With the exception of GSAM commodity index the average correlation has a negative slope. Interestingly the peak in correlations occurs earlier in respect to the other cryptos analysed and has recently turned negative.

Commodities Rolling Correlation with ETH/EUR



It is clear from the chart on the left that ETH/EUR commodity prices tend to behave similarly. Interestingly correlations in this last chart seem to be more volatile. A fifteen-day seasonal pattern seem to emerge, with a mean reverting dynamic.

FITTING LINEAR REGRESSION MODEL

BTC/EUR

In order to forecast the price of BTC, it is assumed that multiple distinct predictor variables(X), like EUR/USD, SPDR Gold Shares and TRC1(coal) might have certain explanatory power of BTC-EUR(y). Thus, the report is interested in assessing the real explanatory power of a set of K candidate variables, by creating a linear regression model with all predictor variables and reducing regressors through a series of procedures. The eventual regression model will help us in forecasting the future price of BTC/EUR.
From a statistical point of view, both the elements of X and y are stochastic processes, namely, collections of random variables, for which we have a set of realizations, Xt and yt; t = 1, …, T. BTC/EUR is analysed over a period of 5 years from 2016/11/01 to 2021/11/05. Explanatory variables are selected from a pool of 21 variables: fiat volume lagged by one day, EUR/USD, USD/JPY, GBP/USD, FXCTEM8 Index, GSAM FX Carry, SPDR Gold Shares, Invesco DB Commodity Index Tracking Fund, TRC1 (coal), CL1 (WTI), Invesco DB Energy Fund, GSAM Commodity Trend, IT10, IT30, JPY10, JPY30, US10, US30, iShares MSCI USA Value Factor ETF, IVW US Equity, VIX and T5YIE.

Model selection

Linearity Test

This step is performed to assess whether the linear regression model should be estimated over prices or returns (logarithmic transformation). Decision is based on adjusted R2, F-stat and p-value of F-stat. Comparing the 2 models statistics summary:

Model	Multiple R-squared	Adjusted R-squared	F-statistic	p-value
RAW	0.9357	0.935	1254	< 2.2e-16
Log	0.9704	0.9699	2151	< 2.2e-16

R-Squared (R^2) is a statistical measure in a regression model that determines the proportion of variance in the dependent variable that is explained by the independent variables. In other words, r-squared shows how well the data fit the regression model (the goodness of fit). While adjusted R-squared is a modified version of R-squared that has been adjusted for the number of predictors in the model. From the model above, we can clearly see that both Multiple R-squared and adjusted R-squared of the linear regression model based on natural logarithm of BTC-EUR are larger than those of raw price. A larger F-statistic and a smaller p-value means the joint effect of all the variables together are significant. From the table above, both the 2 models are significant. Thus, in this case it's more accurate to model the natural logarithm of BTC/EUR price as the outcome y. While in reality when the raw price model performs better in overall regression processes, we might still use the log model, if the raw prices are not normal.

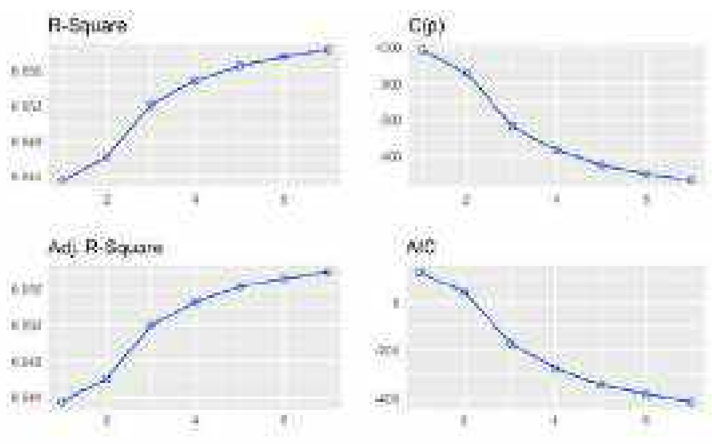
Forward Selection

In order to study the forecasting relationship between a target variable y and a large set of covariates X. A good starting point is to identify the regressor that shows the highest correlation with the target, say x_1: At this point Forward Selection (FWD) consists of regressing y on x_1; storing the residuals $\hat{\varepsilon}1$, and then looking for the covariate in the X information set with the highest correlation with this residual, say x_2.The residual $\hat{\varepsilon}1$is projected onto x_2; a new residual $\hat{\varepsilon}2$ is stored, and the covariate mostly correlated with $\hat{\varepsilon}2$ is next identified. The procedure continues until all the variables in the information set have been ranked, or it can be stopped when a given criterion is satisfied, e.g., the adjusted R2 in a regression of y on the selected regressors is above a given threshold.

The philosophy behind FWD is exactly the opposite of that behind hard thresholding. While the latter can select a large number of regressors very correlated with each other, Forward Selection tends to keep fewer variables, as orthogonal as possible to each other.

After the forward process, the selected 10 predictor variables are as below.

N	Explanatory variable
1	Log (fiat volume lagged by one day)
2	Log (SPDR Gold Shares)
3	Log (iShares MSCI USA Value Factor ETF)
4	Log (IVW US Equity)
5	Log (T5YIE)
6	Log (EUR/USD)
7	Log (JPY30)
8	Log (FXCTEM8 Index)
9	Log (TRC1(coal))
10	Log (IT10)



From R-squared chart, we can see that with increasing number of selected variables from 1 to 10, the eventually R-squared is becoming larger and larger, eventually reaching 0.958. As stated above, the higher the R-squared number, the better current variables fit the regression model (the goodness of fit), which means the better explanation power of current variables X on outcome variable y. Adj-R-squared chart shows exactly the same result as R-squared chart, penalizing the number of regressors.

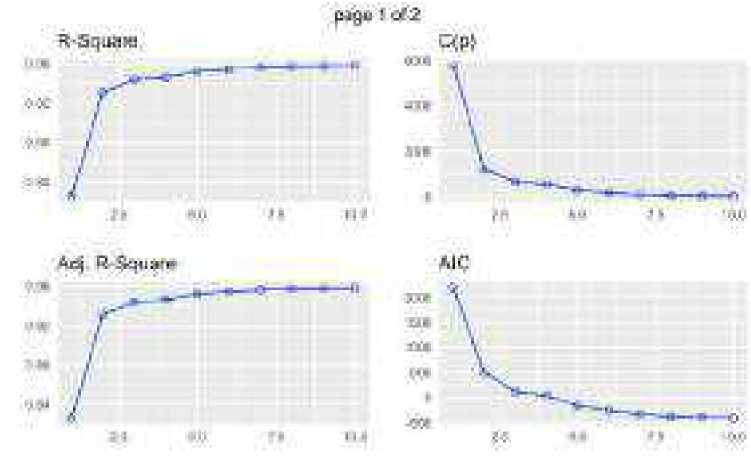
Mallows' Cp-statistic estimates the size of the bias that is introduced into the predicted responses by having an underspecified model. When the Cp value is closer to the number of predictors plus the constant, it means the model is relatively precise and unbiased. From the Cp chart we can clearly see that with number of variables introduced into the model from 1 to 10, the Cp is getting lower to the threshold.

The Akaike information criterion (AIC) is a mathematical method for evaluating how well a model fits the data it was generated from. Lower AIC scores are better, and AIC penalizes models that use more parameters. In this case, we can clearly see that even with more variables introduced, the AIC is decreasing.

Best Subset

Within the forward selected 10 regressors we analyse all the possible best performing subsets and select a preferred model on the basis of the adjusted R2 and of the number of regressors. We deem necessary to produce a parsimonious model that minimizes forecast errors. Below are example best subsets with 5, 6 and 7 variables:

N	Subset with 5 variables	Subset with 6 variables	Subset with 7 variables
1	Log (fiat volume lagged by one day)	Log (fiat volume lagged by one day)	Log (fiat volume lagged by one day)
2	Log (iShares MSCI USA Value Factor)	Log (iShares MSCI USA Value Factor)	Log (iShares MSCI USA Value Factor)
3	Log (IVW US Equity)	Log (IVW US Equity)	Log (IVW US Equity)
4	Log (T5YIE)	Log (T5YIE)	Log (T5YIE)
5	Log (EUR/USD)	Log (EUR/USD)	Log (EUR/USD)
6		Log (JPY30)	Log (JPY30)
7			Log (FXCTEM8 Index)



It is clear from the charts above that increasing the number of variables improves the quality of the model up to a point. From 6 variables to 10 variables, the R-squared only increases by less than 0.005, while the residual errors of forecast increase due to the additional variables introduced. Indeed, numerous explanatory variables are undesirable at least for two reasons. First, the more the regressors in a linear regression model, the less understandable and traceable the model becomes in forecasting future BTC value. Second, considering that some of the predictor variables are only estimates, like 5-Year Breakeven Inflation Rate (T5YIE). More variables also mean more forecast errors in eventual linear regression model. Thus, the selected the model is based on 6-variables best subset.

VIF

In multiple regression, two or more predictor variables might be correlated with each other. This situation is referred as collinearity. There is an extreme situation, called multicollinearity, where collinearity exists between three or more variables even if no pair of variables has a particularly high correlation. This means that there is redundancy between predictor variables. In the presence of multicollinearity, the solution of the regression model becomes unstable. For a given predictor (p), multicollinearity can be assessed by computing a score called the variance inflation factor (or VIF), which measures how much the variance of a regression coefficient is inflated due to multicollinearity in the model. The smallest possible value of VIF is one (absence of multicollinearity). As a rule of thumb, a VIF value that exceeds 5 or 10 indicates a problematic amount of collinearity (James et al. 2014).

When faced to multicollinearity, the concerned variables should be removed, since the presence of multicollinearity implies that the information that this variable provides about the response is redundant in the presence of the other variables (James et al. 2014, P. Bruce and Bruce (2017)). In this step, we test multicollinearity of the selected model (6-variable best subset). Then we eliminate the regressor that breach the threshold with the highest value and recompute it. We stop the process once all the regressor are lower or close to our threshold of 5. These are the 6 selected variables after sub-set process, and their VIF values are as below.

N	Explanatory variables	VIF value
1	Log (fiat volume lagged by one day)	3.96
2	Log (iShares MSCI USA Value Factor ETF)	6.94
3	Log (IVW US Equity)	3.44
4	Log (T5YIE)	4.66
5	Log (EUR/USD)	2.79
6	Log (JPY30)	1.89

VIF=1 means the variable is not correlated with all the others; 1<VIF<5 means considerable collinearity, VIF > 5 is cause for concern and VIF > 10 indicates a serious collinearity problem. Evidently the VIF value of iShares MSCI USA Value Factor ETF has exceeded the threshold 5, resulting it to be eliminated in the final model.

Selected Model Summary

After all the above regressor shrinkage processes, out final regression model is displayed as below.

Coefficients:	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-4.48	0.098	-45.534	<2e-16
Log (fiat volume lagged by one day)	0.28	0.007	38.325	<2e-16
Log (IVW US Equity)	2.10	0.036	57.016	<2e-16
Log (T5YIE)	0.22	0.025	8.655	<2e-16
Log (EUR/USD`)	3.37	0.233	14.434	<2e-16
Log (JPY30)	-0.30	0.024	-12.316	<2e-16

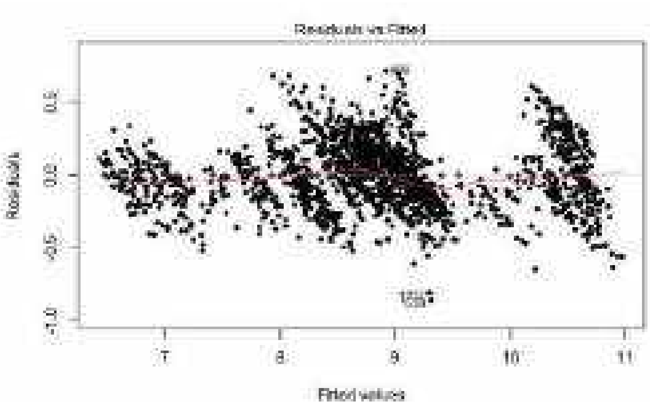
Linear Equation:

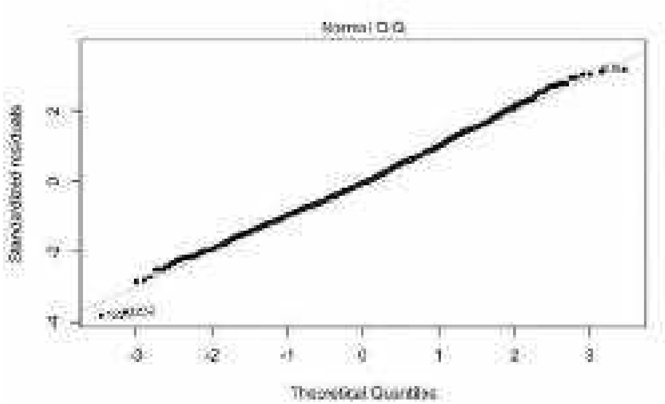
$$\log\,(BTC/EURt) = -4.49 + 0.28 * \ log(fiat\ volume\ lagged\ by\ one\ dayt) + 2.11 * \ log(IVW\ US\ Equityt)\ + 0.22 * \ log(T5YIEt) + 3.38 * \ log(EUR/USDt) + (-0.30) * log(JPY30t)$$

Overall based on the statistics numbers, both Multiple R2 and Adjusted R2 are close to 1, with a larger F-statistic and a smaller p-value, showing that the final model is statistically significant.

The following charts are to test whether the final model satisfies the basic assumptions for linear regression model. Here are four assumptions associated with a linear regression model:

- Linearity: The relationship between X and the mean of Y is linear.
- Homoscedasticity: The variance of residual is the same for any value of X.
- Independence: Observations are independent of each other.
- Normality: The residuals are normally distributed.

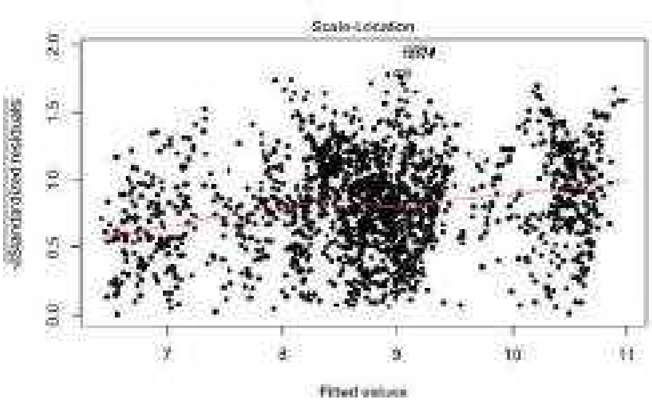




The first plot is used to detect non-linearity, unequal error variances, and outliers. It shows a well-behaved residual and fits values, because：

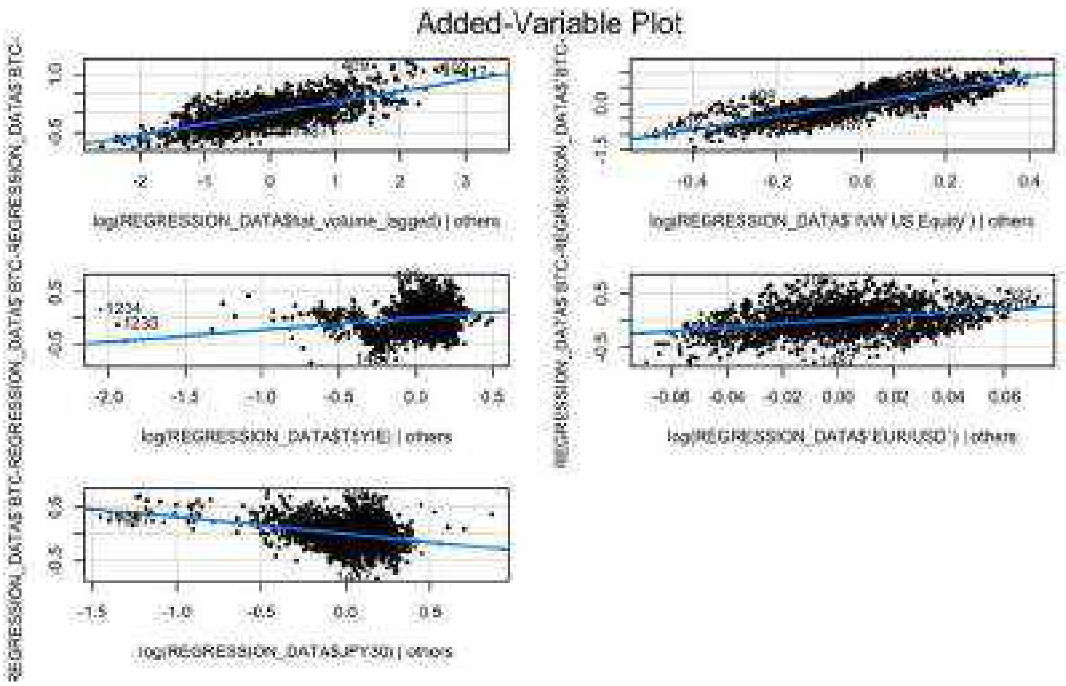
- The residuals "bounce randomly" around the 0 line. This suggests that the assumption that the relationship is linear is reasonable.
- The residuals roughly form a "horizontal band" around the 0 line. This suggests that the variances of the error terms are equal.
- No residual "stands out" from the basic random pattern of residuals. This suggests that there are no large outliers.

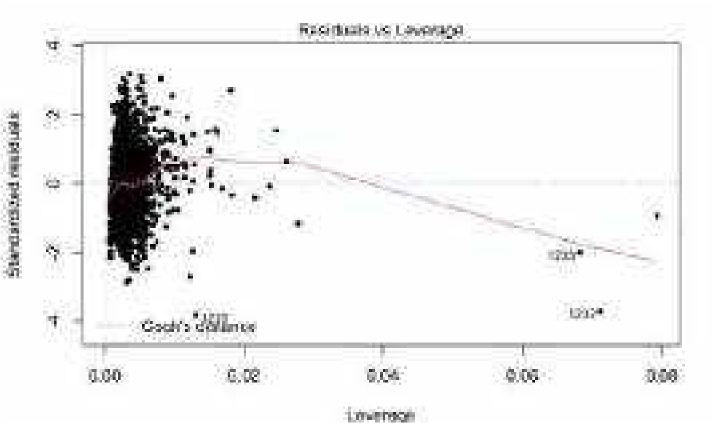
- QQ-norm chart allows us to assess normality of errors. As all the points fall approximately along this reference line, we can assume normality.



Scale-Location plot shows whether residuals are spread equally along the ranges of input variables (predictor). The assumption of equal variance (homoscedasticity) could also be checked with this plot. As we here see a relative horizontal line with randomly spread points, it means that the model is relative accurate.
An added-variable plot is a scatterplot of the transformations of an independent variable (X1) and the dependent variable (y) conditioned to the other independent variables.

These plots allow us to conveniently visualize the relationship between each individual predictor variable and the response variable. For example, in the charts we can find that fiat volume lagged by one day, IVW US Equity and T5YIE are positively related with BTC/EUR, while EUR/USD and JPY 30 are negatively related.
Residuals vs. leverage plot is a type of diagnostic plot that allows us to identify influential observations in a regression model.





This output can also be used to detect heteroskedasticity and non-linearity. The spread of standardized residuals shouldn't change as a function of leverage: here it appears to be relative stable, confirming a linear relationship. Second, points with high leverage may be influential: that is, deleting them would change the model a lot. For this we can look at Cook's distance, which measures the effect of deleting a point on the combined parameter vector. In this case there are no outside Cook's distance, which even can't be observed in the chart, meaning no points have high influence if being deleted.

Model misspecification

Test for homoskedasticity and no correlation

In order to have a model consistent with the assumptions underlying the linear regression model errors should be normally distributed with constant variance.

$$\varepsilon_t \sim N(0; \sigma^2)$$

Focusing the attention on the variance term there are several tests that may be performed to characterize the variance of the errors and to test the presence of homoscedasticity and serial correlation.

White test

The test is based on a Chi-square distribution where (χ_q) where q is the number of regressor present in the linear model. The test is based on the protocol:

$$H_0: \sigma_i^2 = \sigma^2 \;\; H_a: \sigma_i^2 > \sigma^2$$

The result is W=120.42 with a p-value of 4.14e-21. Thus, we reject the null hypothesis highlighting the presence of heteroskedastic errors.

Breusch-Godfrey test

The test is based on a statistic BPG=TR^2 where T represent the sample size and R^2 the coefficient of determination of the regression. The test further characterizes the possibility of heteroskedastic errors through the protocol:

$$H_0: \sigma_i^2 = \sigma^2 \;\; H_a: \sigma_i^2 = \lambda + \delta Z_i$$

Where Z may be possibly the regressor within the model or other non-included explanatory variables. The test investigates the presence of errors correlated to explanatory variables. The result led to BPG= 388.48, p-value < 2.2e-16, thus we reject once again the null hypothesis.

Goldfeld-Quandt test

The test proposes a specific characterization of the relationship between errors and explanatory variables assuming that errors are correlated to the square of the explanatory variable. The test is performed after ranking the observation on the basis of the values of explanatory variables. Then after excluding the 20% central observation the statistic is calculated according to GQ=RSS_2/RSS_1 where 1 and 2 stand for the higher ranked and lower ranked sub samples, the statistic is distributed according to a F distribution. The protocol is:

$$H_0: \sigma_i^2 = \sigma^2 \;\; H_a: \sigma_i^2 = cz_i^2$$

We test this statistic by ordering our samples on the basis of different explanatory variables.

Ranking variable	Fiat Volume lagged	IVW US Equity	T5YIE	EUR/USD	JPY30
GQ statistic	1.50	0.88	1.50	1.55	1.49
p-value	2.15e-08	0.94	2.27e-08	1.39e-09	3.97e-08

Durbin Watson test

The test analyses the presence of serial correlation in the errors. The statistic should return a value around 2 if DW statistic departs significantly from this value there is likely presence of serial correlation in the model. The protocol for this test is:

$$H_0: \varepsilon_i\ uncorrelated\ \ H_a: \varepsilon_i = \rho\varepsilon_{i-1} + u_i$$

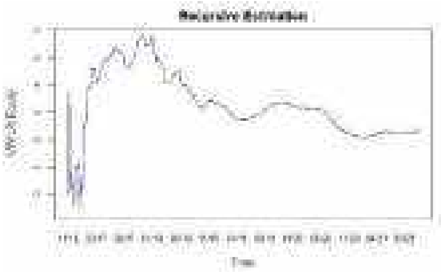
The result led to DW=0.41, thus we reject the null hypothesis.

Test for parameter instability

Another implicit assumption in linear regression models is stability of model parameters. However, when modelling timeseries is it common to encounter parameter instability due to structural changes in the explanatory variables, significantly decreasing the precision of the model. Thus, an analysis is performed in order to assess the presence of parameter instability.

Recursive estimation

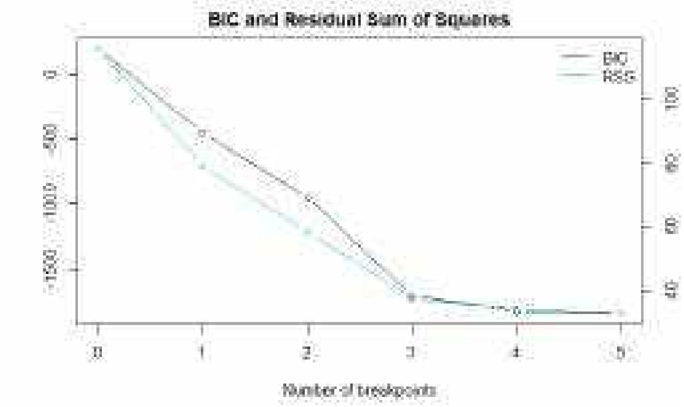
To assess the presence of parameter instability we first use the method of recursive estimator. This method estimates recursively the linear parameters progressively enlarging the sample size under consideration. Thus, recursive estimation allows us to produce a very effective graphical output to observe the potential presence of parameter instability. The charts below show the parameters that are more clearly not sable over time.



Bai-Perron test

This test statistically analyses the presence of structural breaks. The test is distributed according to an F statistic. Bai Perron test allows use to map the number of structural breaks in the model and simultaneously provides us estimates on the performance of the model if these structural breaks are considered within the regression. We cap the number of structural breaks to five since we deem necessary a parsimonious model. The protocol of this test is:

$$H_0: no\ breaks\ \ H_a: m\ breaks$$



As it is clear from the chart above the performance of the model significantly improves if we consider 4 structural breaks. The structural break dates are picked on the basis of RSS minimization and are calculated by the R software. These are: 14/01/2018, 04/04/2019, 25/01/2020, 14/11/2020.

Chow test

In order to further test the significance of these break dates we perform the Chow test. This test compares two models based on the two different subsamples created by each break date with the model estimated over the whole sample. The test is distributed according to an F stat. The protocol is:

$$H_0: \beta_1 = \beta_2 \; H_a: \; \beta_1 \neq \beta_2$$

Where 1 and 2 are the two subsamples built on the basis of each break date. The result of the test for each break date are summarized in the table below.

Break date	14/01/2018	04/04/2019	25/01/2020	14/11/2020
CH statistic	79.47	108.2	123.21	49.92
p-value	2.2e-16	2.2e-16	2.2e-16	2.2e-16

Dummy Variables

In order to counter the presence of parameter stability dummy variables according to previously tested break dates are included in the model. In the table below are stated the main statistic of the new model with the addition of dummy variables to counter parameter instability.

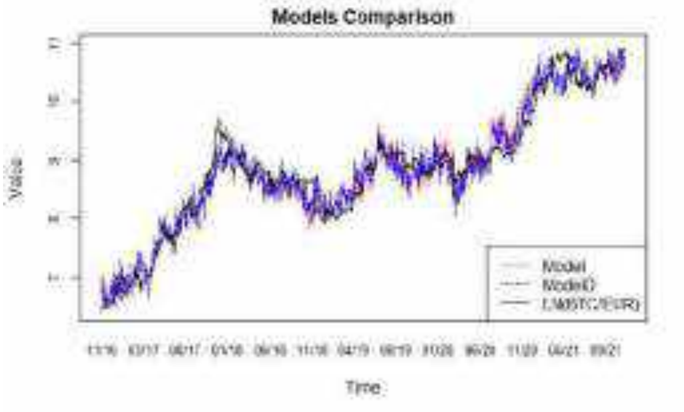
Coefficients	Estimate	Std. Error	t value	PR(>|t|)
(Intercept)	-4.708	0.326	-14.40	< 2e-16
Fiat Volume lagged	0.279	0.007	38.78	< 2e-16
IVW US Equity	2.248	0.103	21.79	< 2e-16
T5YIE	0.103	0.031	3.25	0.001
EUR/USD	3.047	0.246	12.38	< 2e-16
JPY30	-0.270	0.075	-3.57	0.0003
dummy1	0.022	0.024	0.91	0.359
dummy2	0.105	0.039	2.69	0.007
dummy3	-0.228	0.029	-7.74	1.60E-14
dummy4	0.067	0.036	1.85	0.064

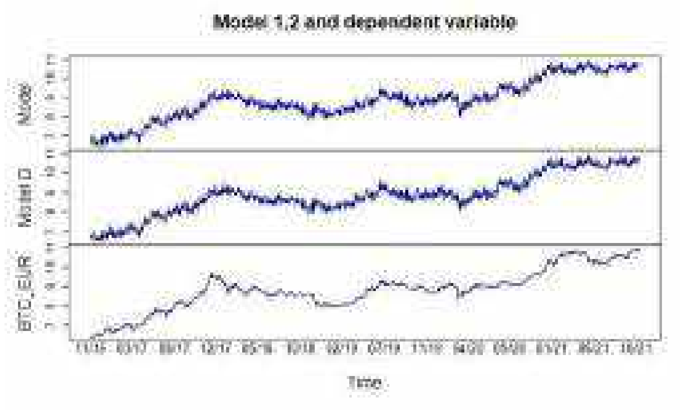
Multiple R-squared	Adjusted R-squared	F-statistic	p-value
0.9449	0.9446	3469	2.2e-16

Models' comparison

Graphical Output

To complete the analysis two graphical output are displayed to assess on a visual basis the performance of the model both with and without dummies in respect to our independent variable (ln(BTC/EUR)).





Diebold Mariano test

We also assess the relative performance of the two models comparing them with the Diebold Mariano statistic. This statistic is particularly useful since it does not assumes any particular distribution of errors and it is flexible since it allows the comparison with several loss functions. We calculated this statistic with two different loss function mean squared errors and absolute errors. The protocol of the test is:

$$H_0: model\ is\ accurate\ as\ model\ with\ dummies\ H_a: model\ is\ less\ accurate\ than\ model\ with\ dummies$$

The result for both MSE and MAE are summarized in the table below.

Loss Function	DM statistic	p-value
MSE	4.1775	1.474e-05
MAE	2.0633	0.01954

Conclusion

The introduction of dummies clearly enhances the model ability to predict the independent variable. Although the model has some limitations it allows us to draw some significant conclusions:

1. The independent variable is extremely volatile; thus, the introduction of dummies variables clearly enhances the model performance. This can be clearly seen through the Diebold Mariano statistic; the model with dummies performs significantly better if we use MSE as loss functions that tend to penalize larger prediction errors.
2. The returns of BTC/EUR seem highly correlated with the return of the index IVW US Equity. This index is composed by US growth equities with an average P/E of 42.90 as reported by BlackRock. Thus, bitcoin is confirmed to be a highly risky asset, strongly correlated with growth equities.
3. Bitcoin returns are only slightly correlated with 5-Year Breakeven Inflation Rate. The regressor has a coefficient of 0.103. Thus, Bitcoin as inflation hedge is questionable.
4. Bitcoin returns are negatively correlated with Japanese long term bond returns, this is consistent with the interpretation of bitcoin as risky asset.
5. Bitcoin returns are significantly correlated with EUR/USD return, the pair is considered a good parameter of risk-on/risk-off, performing well in high risk tolerance environments.
6. Change in fiat volume lagged by one day are consistently correlated with bitcoins returns, thus liquidty based trading strategies may prove profitable.

BTC/USD

The procedures are exactly the same as what's done above, except that this model is used to forecast the future price of BTC-USD(y) after assessing the explanatory power of the 21 predictor variables(X) and reducing regressors.

Model selection

Linearity Tests

First compare the performance of linear regression model based on prices and log returns.

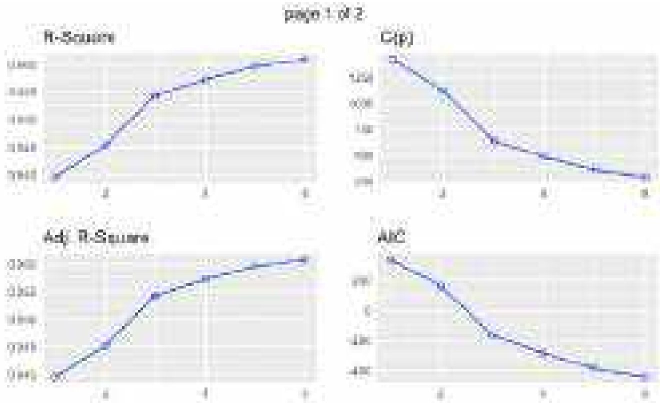
Model	Multiple R-squared	Adjusted R-squared	F-statistic	p-value
RAW	0.9409	0.9403	1372	< 2.2e-16
Log	0.9727	0.9722	2336	< 2.2e-16

From the table, we can clearly see that both Multiple R-squared and adjusted R-squared of the linear regression model based on natural logarithm of BTC-USD are larger than those of raw price. While in terms of F-statistics and p-value, all variables in both models are significant. Thus, in this case, it's more accurate to model the natural logarithm of BTC-USD price as the outcome y.

Forward Selection

After the forward process, the selected 8 predictor variables are as below.

N	Explanatory variable
1	Log (fiat volume lagged by one day)
2	Log (iShares MSCI USA Value Factor ETF)
3	Log (IVW US Equity)
4	Log (T5YIE)
5	Log (EUR/USD)
6	Log (JPY30)
7	Log (FXCTEM8 Index)
8	Log (USD/JPY)



From R-squared and Adj R-squared charts, we can see that with increasing number of selected variables from 1 to 8, the eventually number is becoming larger, eventually reaching 0.960, which shows the best explanation power of BTC-USD is based on 8 selected variables.

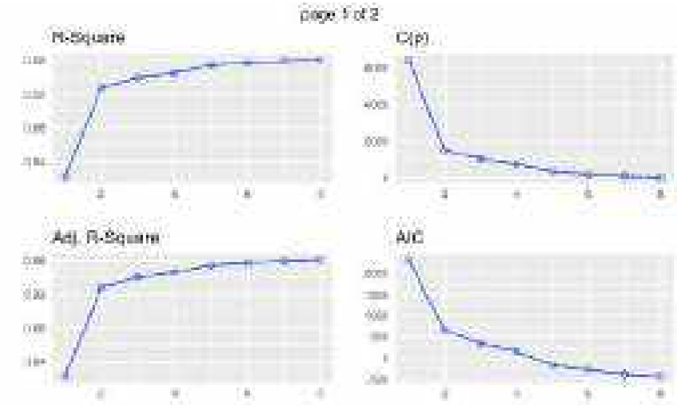
From the Cp chart we can clearly see that with number of variables introduced into the model from 1 to 8, the Cp is getting closer to the number of predictors plus the constant, showing the model is getting more precise and unbiased.

In the last chart, a decreasing AIC score with more variables shows the linear regression model is performing better and better.

Best Subset

Here we analyse all the possible best performing subsets with the forward selected 8 regressors. Below are example best subsets with 4 and 5 variables:

N	Subset with 4 variables	Subset with 5 variables
1	Log (fiat volume lagged by one day)	Log (fiat volume lagged by one day)
2	Log (iShares MSCI USA Value Factor ETF)	Log (iShares MSCI USA Value Factor ETF)
3	Log (IVW US Equity)	Log (IVW US Equity)
4	Log (EUR/USD)	Log (T5YIE)
5		Log (EUR/USD)



It is clear from the charts above that increasing the number of variables improves the quality of the model up to a point. From 5 variables to 8 variables, the R-squared only increases by less than 0.005, while the residual errors of forecast increase due to the additional variables introduced. Thus, the selected the model is based on 5-variables best subset.

VIF

These are the 5 selected variables after sub-set process, and their VIF values are as below:

N	Explanatory variables	VIF Value
1	Log (fiat volume lagged by one day)	4.12
2	Log (iShares MSCI USA Value Factor ETF)	6.51
3	Log (IVW US Equity)	3.57
4	Log (T5YIE)	3.72
5	Log (EUR/USD)	2.24

Clearly VIF of Log (iShares MSCI USA Value Factor ETF) has exceeded the threshold of 5, meaning it has strong multicollinearity with 2 or more variables. So, it has to be eliminated from the final model.

Selected Model Summary

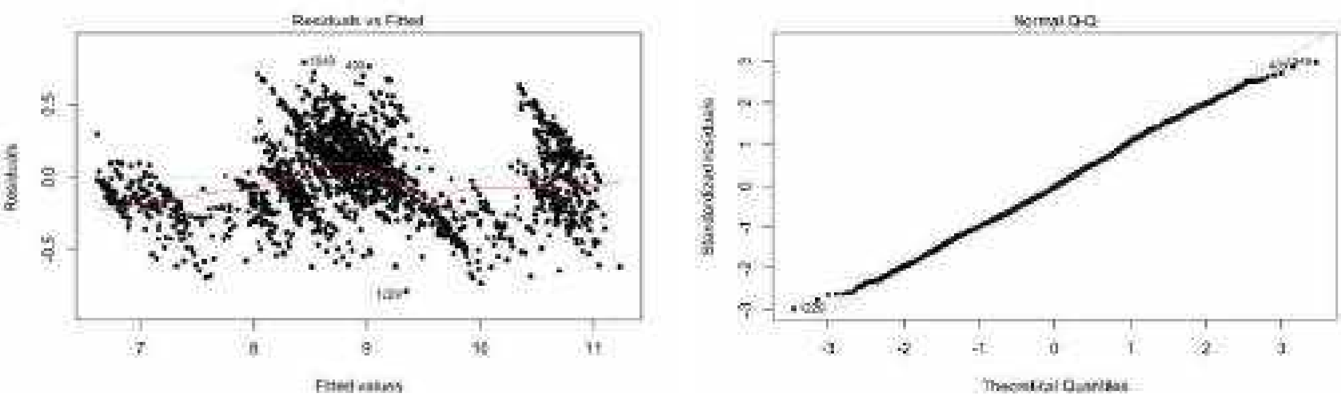
Coefficients:	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-5.48	0.100	-54.637	<2e-16
Log (fiat volume lagged by one day)	0.31	0.008047	38.681	<2e-16
Log (IVW US Equity)	2.16	0.039	54.703	<2e-16
Log (T5YIE)	0.01	0.024	0.774	0.439
Log (EUR/USD)	3.41	0.219	15.53	<2e-16

Model	Multiple R-squared	Adjusted R-squared	F-statistic	p-value
Final Log	0.9391	0.9389	7035	< 2.2e-16
Log	0.9727	0.9722	2336	< 2.2e-16

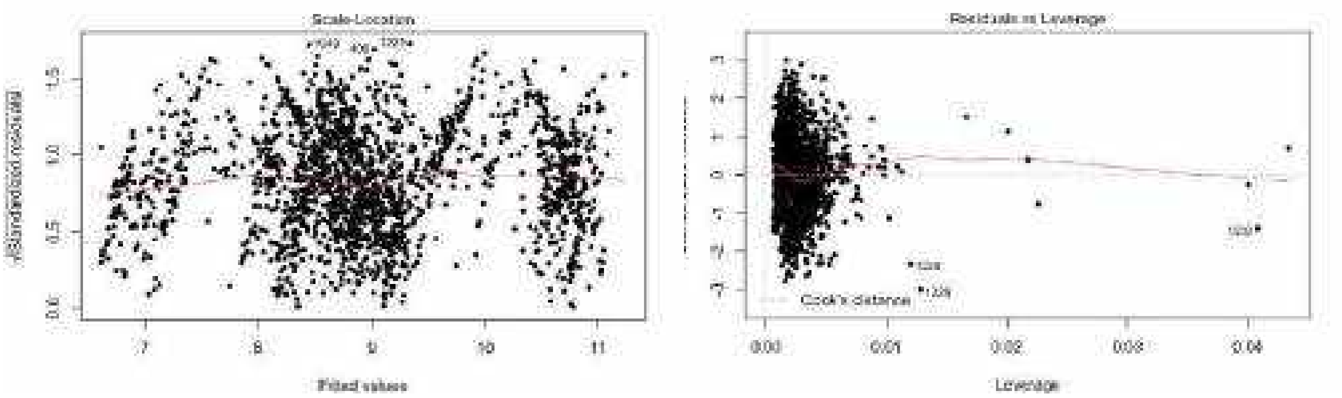
Linear Equation:

$$\log(BTC/USDt) = -5.48 + 0.31 * log(fiat\ volume\ lagged\ by\ one\ dayt) + 2.16 * log(IVW\ US\ Equityt) + 0.02 * log(T5YIEt) + 3.42 * log(EUR/USDt)$$

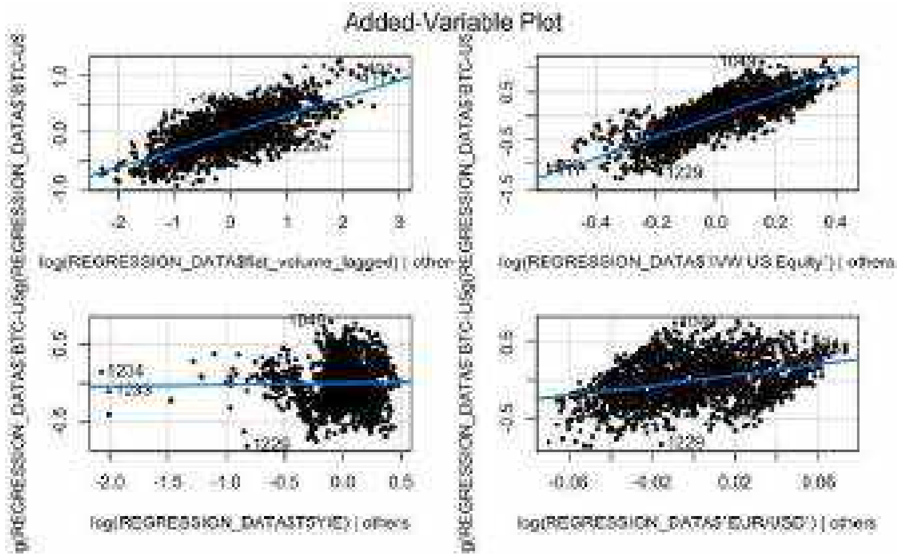
Overall based on the statistics numbers, both Multiple R2 and Adjusted R2 are close to 1, with a larger F-statistic and a smaller p-value, showing that the final model is statistically significant.



- The residuals& fitted plot shows a well-behaved result, because:
- The residuals "bounce randomly" around the 0 line. This suggests that the assumption that the relationship is linear is reasonable.
- The residuals roughly form a "horizontal band" around the 0 line. This suggests that the variances of the error terms are equal.
- No residual "stands out" from the basic random pattern of residuals. This suggests that there are no large outliers.
- As all the points fall approximately along this reference line, we can assume normality.



As we here see a relative horizontal line with randomly spread points, it means that the model is relative accurate and of homoscedasticity. The plot shows that the spread of standardized residuals appears to be relative stable, confirming a linear relationship; there are no points that have high influence on the model if being deleted, with no points outside Cook's distance, which even can't be observed in the chart.



These plots allow us to conveniently visualize the relationship between each individual predictor variable and the response variable. For example, in the charts we can find all the 4 variables are positively related with BTC-USD, while fiat volume lagged by one day and IVW US Equity shows stronger relationship.

Model misspecification

Test for homoskedasticity and no correlation

White test

$$H_0: \sigma_i^2 = \sigma^2 \;\; H_a: \sigma_i^2 > \sigma^2$$

The result is W= 317.77 with a p-value of 2.70e-62. Thus, we reject the null hypothesis highlighting the presence of heteroskedastic errors.

Breusch-Godfrey test

$$H_0: \sigma_i^2 = \sigma^2 \;\; H_a: \sigma_i^2 = \lambda + \delta Z_i$$

The result led to BPG= 164.94, p-value < 2.2e-16, thus we reject once again the null hypothesis.

Goldfeld-Quandt test

$$H_0: \sigma_i^2 = \sigma^2 \;\; H_a: \sigma_i^2 = c z_i^2$$

We test this statistic by ordering our samples on the basis of different explanatory variables.

Ranking variable	Fiat Volume lagged	IVW US Equity	T5YIE	EUR/USD	ISHARES VALUE
GQ statistic	1.61	0.77	1.10	1.01	0.97
p-value	5.9e-11	0.99	0.08	0.44	0.63

Durbin Watson test

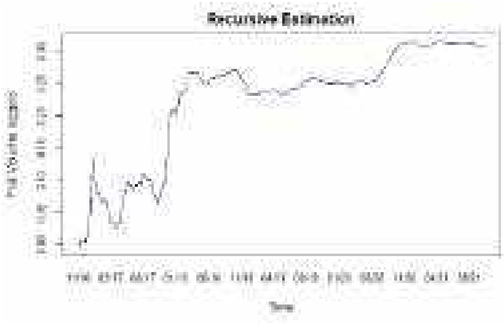
$$H_0: \varepsilon_i \; uncorrelated \;\; H_a: \varepsilon_i = \rho \varepsilon_{i-1} + u_i$$

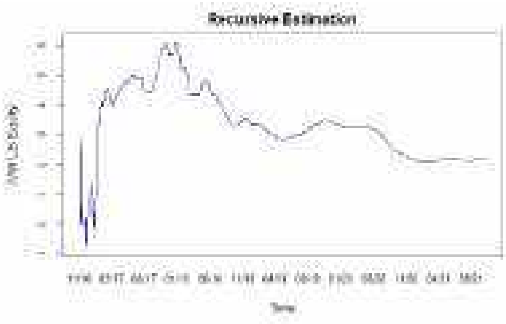
The result led to DW= 0.3442366, thus we reject the null hypothesis.

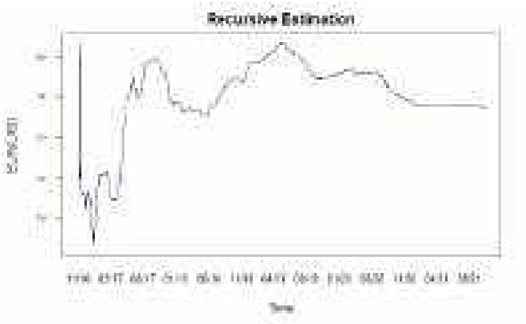
Test for parameter instability

Recursive estimation

The charts below show the parameters that are more clearly not sable over time.

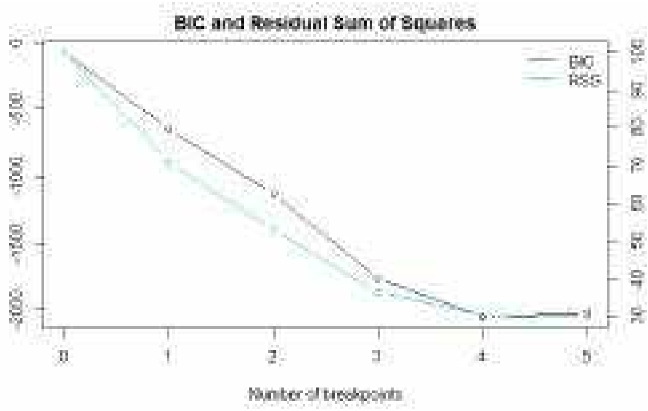






Bai-Perron test

$$H_0: no\ breaks\ \ H_a: m\ breaks$$



As it is clear from the chart above the performance of the model significantly improves if we consider 3 structural breaks. These are 15/01/2018, 04/04/2019, 04/03/2020.

Chow test

$$H_0: \beta_1 = \beta_2\ H_a: \ \beta_1 \neq \beta_2$$

Break date	15/01/2018	04/04/2019	04/03/2020
CH statistic	83.85	91.2	114.56
p-value	2.2e-16	2.2e-16	2.2e-16

Dummy Variables

In the table below are stated the main statistic of the new model with the inclusion of dummy variables to counter parameter instability.

Coefficients	Estimate	Std. Error	t value	PR(>|t|)
(Intercept)	-12.405	0.475	-26.07	< 2e-16
Fiat Volume lagged	0.257	0.007	33.94	< 2e-16
IVW US Equity	1.090	0.117	9.26	< 2e-16
T5YIE	-0.335	0.031	-10.53	< 2e-16
EUR/USD	4.175	0.225	18.53	< 2e-16
ISHARES VALUE	2.794	0.163	17.04	< 2e-16

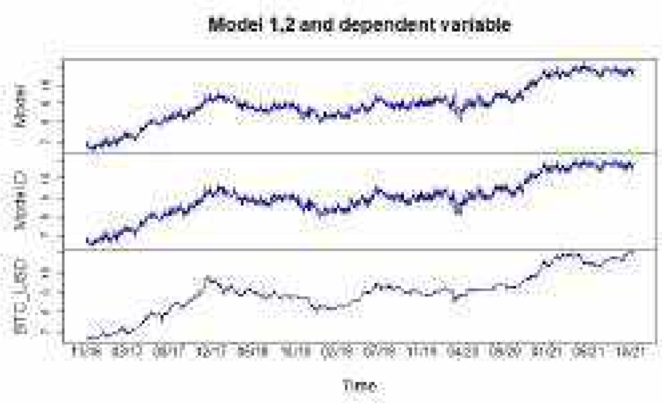
dummy1	0.114	0.020	5.68	1.57E-08
dummy2	0.156	0.029	5.37	8.55E-08
dummy3	0.099	0.033	2.94	0.003

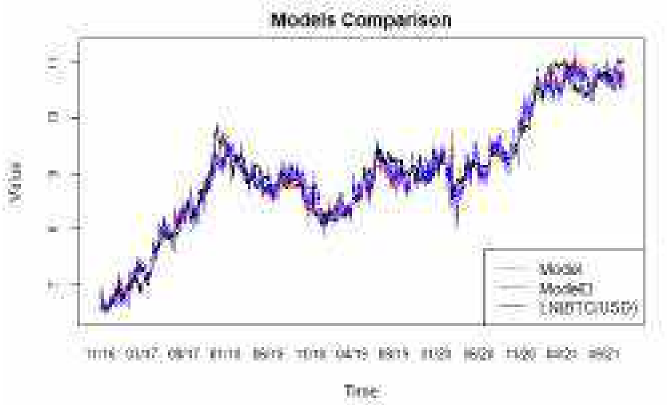
Multiple R-squared	Adjusted R-squared	F-statistic	p-value
0.9544	0.9542	4772	2.2e-16

Models' comparison

Graphical Output

To complete the analysis two graphical output are displayed to assess on a visual basis the performance of the model both with and without dummies in respect to our independent variable (ln(BTC/USD)).





Diebold Mariano test

$$H_0: model\ is\ accurate\ as\ model\ with\ dummies\ H_a:\ \ model\ is\ more\ accurate\ than\ model\ with\ dummies$$

Loss Function	DM statistic	p-value
MSE	1.20	0.886
MAE	0.33	0.629

Conclusion

The introduction of dummies clearly enhances the model ability to predict the independent variable. While the model has some limitations it allows us to draw some significant conclusions:

1. Although the model with dummies increases the quality of forecast, dummies are not as performing as in the first model analysed. Indeed, while the model with dummies is more accurate, dummies fail to pass the Diebold Mariano standard test. This test explicitly assess through the alternative hypothesis *model is less accurate than model with dummies.*
2. The returns of BTC/USD are less correlated with returns of IVW US Equity than in the model previously analysed. The underlying reason may rest in the presence of another equity index ISHARES VALUE. The index is highly concentrated in technology and financial sectors thus our previous conclusion is still relevant.
3. In this new model Bitcoin returns are negatively correlated with 5-Year Breakeven Inflation Rate. The regressor has a coefficient of -0.335. Thus, this further disprove Bitcoin as an inflation hedging asset.
4. Bitcoin returns are still significantly correlated with EUR/USD return, the pair is considered a good parameter of risk on-risk off, performing well in higher risk tolerance environments.

ETH/EUR

The procedures are exactly the same as what's done above, except that this model is used to forecast the future price of ETH-EUR (y) after assessing the explanatory power of the 21 predictor variables(X) and reducing regressors.

Model selection

Linearity Tests

First compare the performance of linear regression model based on prices and log returns.

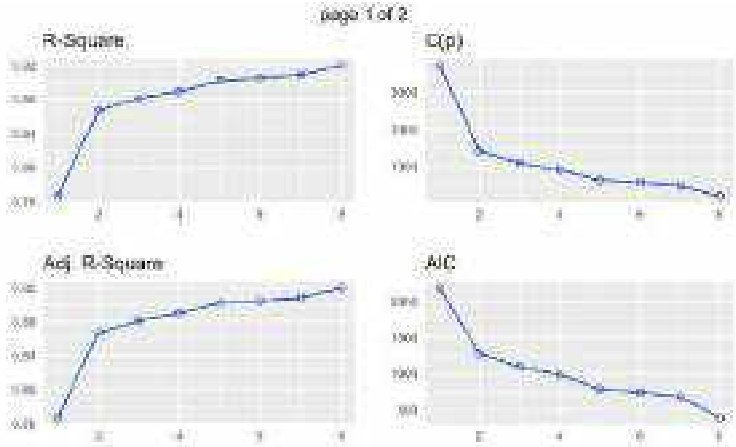
Model	Multiple R-squared	Adjusted R-squared	F-statistic	p-value
RAW	0.9543	0.9537	1594	< 2.2e-16
Log	0.9326	0.9315	783.3	< 2.2e-16

Here is case when the raw price model performs better in overall regression processes, with both higher R-squared and adjusted R-squared multiples, we still use the log model, because for linear regression model because the raw prices are not normal.

Forward Selection

After the forward process, the selected 9 predictor variables are as below.

N	Explanatory variable
1	Log (fiat volume lagged by one day)
2	Log (VIX)
3	Log (GBP/USD)
4	Log (GSAM Commodity Trend)
5	Log (JPY30)
6	Log (JPY10)
7	Log (EUR/USD)
8	Log (USD/JPY)
9	Log (TRC1(coal))



From R-squared and Adj R-squared charts, we can see that with increasing number of selected variables from 1 to 8, the eventually number is becoming larger, eventually reaching 0.92, which shows the best explanation power of ETH-EUR is based on 8 selected variables.

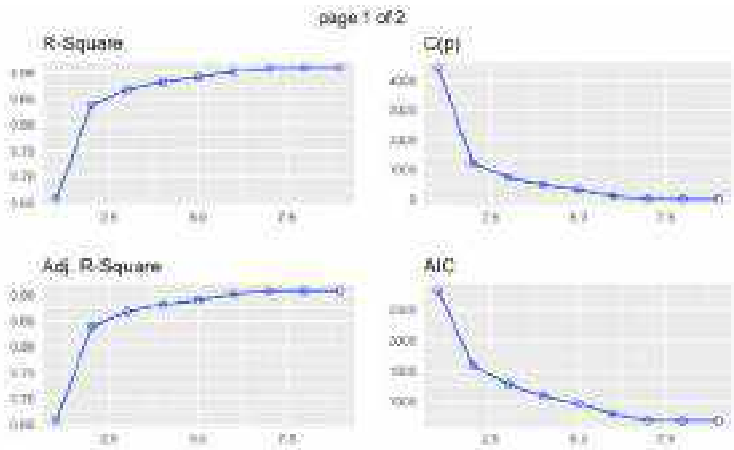
From the Cp chart we can clearly see that with number of variables introduced into the model from 1 to 8, the Cp is getting closer to the number of predictors plus the constant, showing the model is getting more precise and unbiased. In the last chart, a decreasing AIC score with more variables shows the linear regression model is performing better and better.

Best Subset

Here we analyse all the possible best performing subsets with the forward selected 8 regressors.
Below are example best subsets with 4 and 5 variables:

N	Subset with 5 variables	Subset with 6 variables
1	Log (fiat volume lagged by one day)	log (fiat volume lagged by one day)

2	Log (GSAM Commodity Trend)	Log (GSAM Commodity Trend)
3	Log (VIX)	Log (VIX)
4	Log (EUR/USD)	Log (GBP/USD)
5	Log (TRC1(coal))	Log (TRC1(coal))
6		Log (USD/JPY)



It is clear from the charts above that increasing the number of variables improves the quality of the model up to a point. From 6 variables to 9 variables, the R-squared only increases by less than 0.005, while the residual errors of forecast increase due to the additional variables introduced.
Thus, the selected the model is based on 6-variables best subset.

VIF

These are the 6 selected variables after sub-set process, and their VIF values are as below:

N	Explanatory variables	VIF Value
1	Log (fiat volume lagged by one day)	2.38
2	Log (GSAM Commodity Trend)	3.49
3	Log (VIX)	2.32
4	Log (GBP/USD)	2.04
5	Log (TRC1(coal))	3.01
6	Log (USD/JPY)	1.69

No VIF values have exceeded the threshold of 5, meaning no variable has strong multicollinearity with 2 or more variables. So, no variable has to be eliminated from the final model.

Selected Model Summary

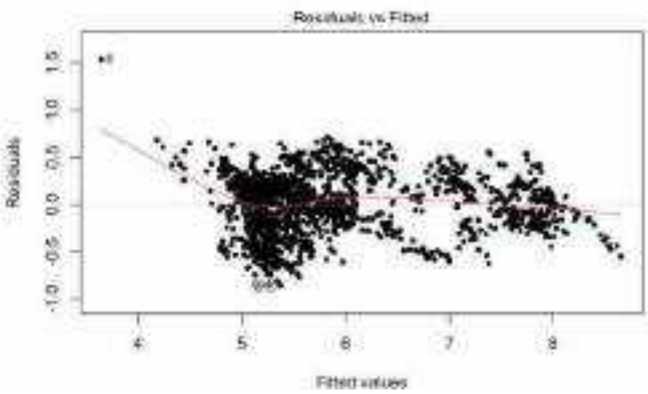
Coefficients:	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-35.16	2.454	-14.33	<2e-16
Log (fiat volume lagged by one day)	0.11	0.007	15.29	<2e-16
Log (GSAM Commodity Trend)	12.18	0.401	30.34	<2e-16
Log (GBP/USD)	6.66	0.272	24.48	<2e-16
Log (VIX)	-0.51	0.030	-16.95	<2e-16
Log (USD/JPY)	-5.86	0.412	-14.21	<2e-16
Log (TRC1(coal))	1.09	0.060	18.06	<2e-16

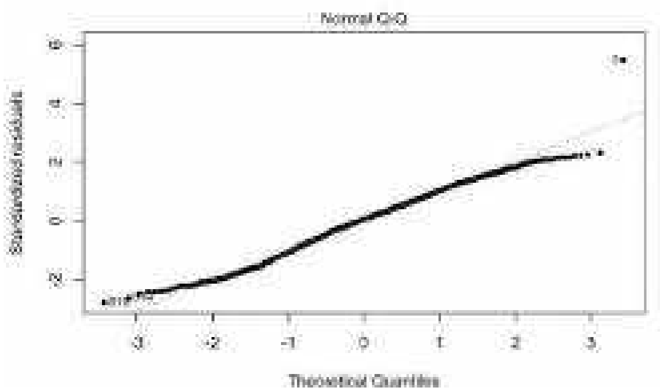
Model	Multiple R-squared	Adjusted R-squared	F-statistic	p-value
Final Log	0.9017	0.9013	2477	< 2.2e-16
Log	0.9326	0.9315	783.3	< 2.2e-16

Linear Equation:

$$\log(ETH/EURt) = -35.17 + 0.11 * log(fiat\ volume\ lagged\ by\ one\ dayt) + 12.18 * log(\text{GSAM Commodity Trend}t) + 6.67 * log(GBP/USDt) + (-0.51) * log(VIXt) + (-5.86) * log(USD/JPYt) + 1.10 * log(\text{TRC1(coal)}t)$$

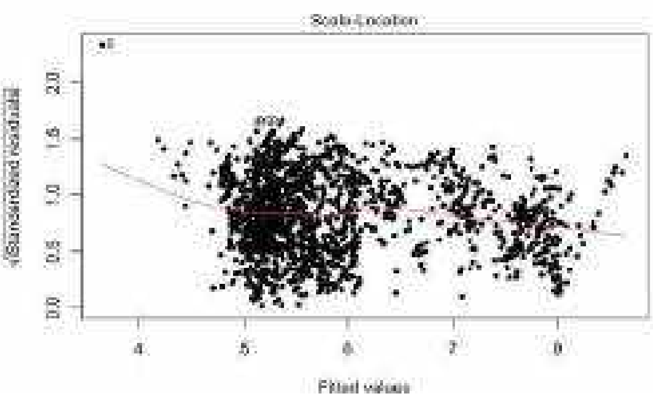
Overall based on the statistics numbers, both Multiple R2 and Adjusted R2 are close to 1, with a larger F-statistic and a smaller p-value, showing that the final model is statistically significant.

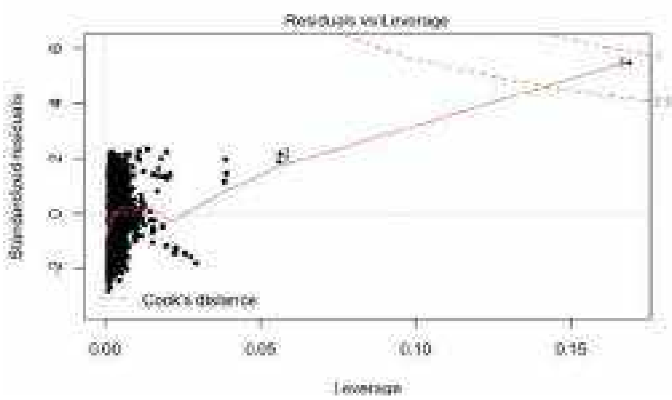




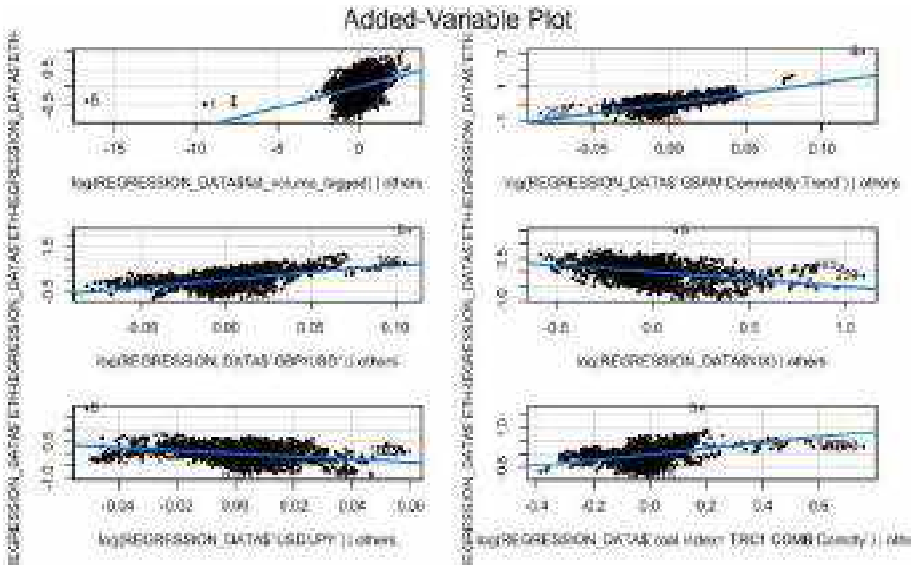
The residuals and fitted plot show a relatively well-behaved result, because:

- The residuals "bounce randomly" around the 0 line. This suggests that the assumption that the relationship is linear is reasonable.
- The residuals roughly form a "horizontal band" around the 0 line except for fitted values at 4 or 5. This suggests that the variances of the error terms are relative equal.
- 1 residual at 5 "stands out" from the basic random pattern of residuals. This suggests that there are roughly no large outliers.
- As all the points fall approximately along this reference line, we can assume normality.





As we here see a relative horizontal line with randomly spread points, it means that the model is relative accurate and of homoscedastic. The last plot shows, the spread of standardized residuals appears to be relative stable, confirming a linear relationship; there is 1 point that has high influence on the model if being deleted, because it's outside Cook's distance, which is the point with residual at the fifth observation.



These plots allow us to conveniently visualize the relationship between each individual predictor variable and the response variable. For example, in the charts we can find all, but USD/JPY, are positively related with ETH/EUR.

Model misspecification

Test for homoskedasticity and no correlation

White test

$$H_0: \sigma_i^2 = \sigma^2 \quad H_a: \sigma_i^2 > \sigma^2$$

The result is W= 446.75 with a p-value of 4.601e-88. Thus, we reject the null hypothesis highlighting the presence of heteroskedastic errors.

Breusch-Godfrey test

$$H_0: \sigma_i^2 = \sigma^2 \quad H_a: \sigma_i^2 = \lambda + \delta Z_i$$

The result led to BPG= 195.17, p-value < 2.2e-16, thus we reject once again the null hypothesis.

Goldfeld-Quandt test

$$H_0: \sigma_i^2 = \sigma^2 \quad H_a: \sigma_i^2 = c z_i^2$$

We test this statistic by ordering our samples on the basis of different explanatory variables.

Ranking variable	Fiat Volume lagged	GBP/USD	GSAM Commodity trend	USD/JPY	VIX
GQ statistic	0.75	1.29	0.46	1.19	2.41
p-value	0.99	0.0005	1	0.01	2.2e-16

Durbin Watson test

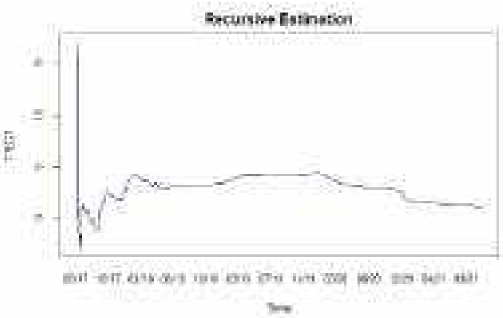
$$H_0: \varepsilon_i \; uncorrelated \quad H_a: \varepsilon_i = \rho\varepsilon_{i-1} + u_i$$

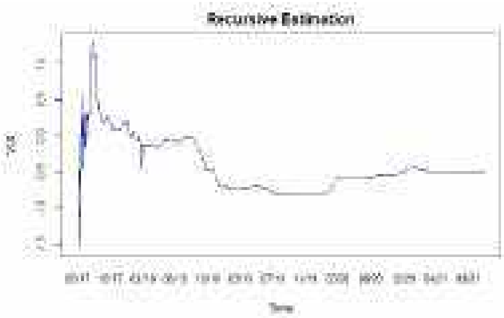
The result led to DW= 0.13, thus we reject the null hypothesis.

Test for parameter instability

Recursive estimation

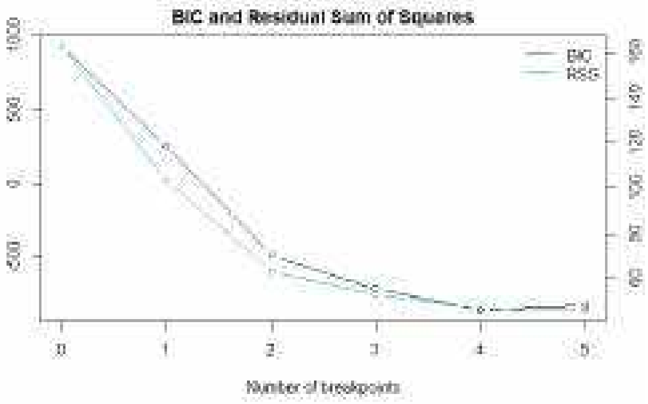
The charts below show the parameters that are clearly not sable over time.





Bai-Perron test

$$H_0: no\ breaks\quad H_a: m\ breaks$$



As it is clear from the chart above the performance of the model significantly improves if we consider 3 structural breaks. These are 04/09/2018, 01/02/2020, 23/01/2021.

Chow test

$$H_0: \beta_1 = \beta_2\ H_a: \ \beta_1 \neq \beta_2$$

Break date	04/09/2018	01/02/2020	23/01/2021
CH statistic	129.06	125.62	22.26
p-value	2.2e-16	2.2e-16	2.2e-16

Dummy Variables

In the table below are stated the main statistic of the new model with the inclusion of dummy variables to counter parameter instability.

Coefficients	Estimate	Std. Error	t value	PR(>|t|)
(Intercept)	14.563	2.990	4.871	1.22E-06
Fiat Volume lagged	0.120	0.005	21.09	< 2e-16
GBP/USD	2.131	0.242	8.77	< 2e-16
GSAM Commodity trend	2.332	0.591	3.94	8.34E-05
TRC1	1.271	0.047	27.00	< 2e-16
USD/JPY	-6.462	0.322	-20.02	< 2e-16
VIX	-0.012	0.0009	-13.07	< 2e-16
dummy1	-0.667	0.018	-36.93	< 2e-16

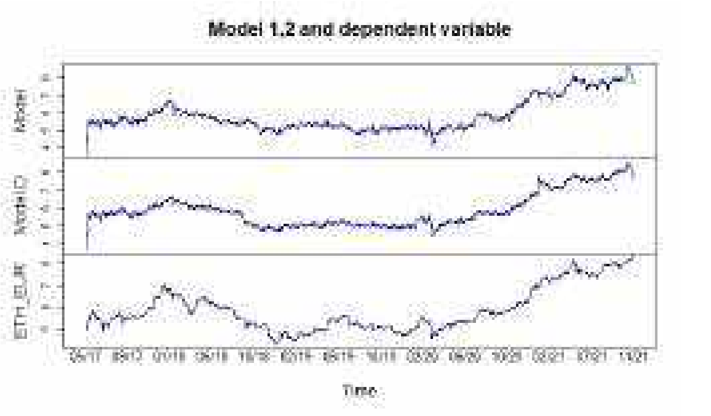
dummy2	0.356	0.029	12.06	< 2e-16
dummy3	0.977	0.041	23.50	< 2e-16

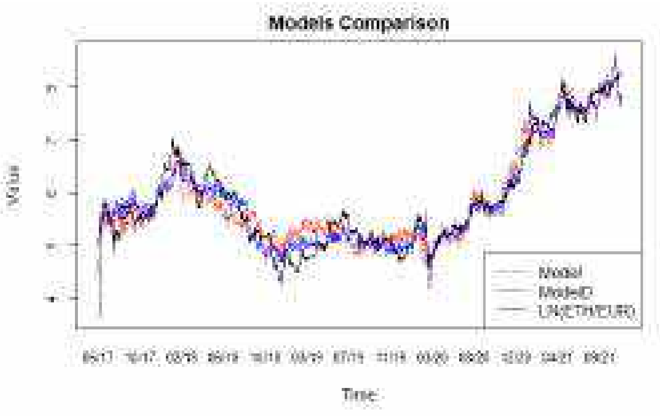
Multiple R-squared	Adjusted R-squared	F-statistic	p-value
0.945	0.9447	3089	2.2e-16

Models' comparison

Graphical Output

To complete the analysis two graphical output are displayed to assess on a visual basis the performance of the model both with and without dummies in respect to our independent variable (ln(ETH/EUR)).





Diebold Mariano test

$$H_0: model\ is\ accurate\ as\ model\ with\ dummies\ H_a:\ \ model\ is\ less\ accurate\ than\ model\ with\ dummies$$

Loss Function	DM statistic	p-value
MSE	16.68	2.2e-16
MAE	17.29	2.2e-16

Conclusion

The introduction of dummies clearly enhances the model ability to predict the independent variable. Although the model has some significant limitations it allows us to draw some meaningful conclusions:

1. In this model the addition of dummies clearly enhances the forecast quality. This is both true at MAE and MSE level, therefore it does not depend mainly on the presence of large outliers.
2. The returns of ETH/EUR are significantly correlated with GBP/USD, with a coefficient of 2.13. This comes as no surprise, since pound in the last years has become a currency with significant volatility due to higher interest rates, lower unemployment and Brexit.
3. Ethereum returns are significantly correlated with GSAM Commodity trend an equal risk weighted exposure to a portfolio containing Metals, Energy and Agro industries Sector Trend strategies.
4. Ethereum is also positively correlated with TRC1, with a beta of 1.271. TRC1 is an index based on generic thermal coal futures. This seems to support the idea that energy prices, and in particular coal (the number one energy source in China) affect crypto values.
5. Quite surprisingly VIX has a low coefficient. Thus, we cannot affirm a strong relationship between US equities volatility and Ethereum.

ETH/USD

The procedures are exactly the same as what's done above, except that this model is used to forecast the future price of ETH-USD (y) after assessing the explanatory power of the 21 predictor variables(X) and reducing regressors.

Model selection

Linearity Tests

First compare the performance of linear regression model based on prices and log returns.

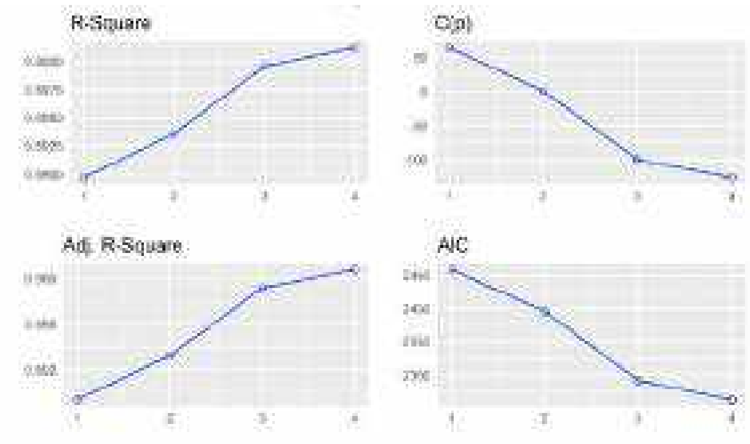
Model	Multiple R-squared	Adjusted R-squared	F-statistic	p-value
RAW	0.02205	0.0107	1.943	0.006363
Log	0.912	0.9106	680.3	< 2.2e-16

From the table, we can clearly see that both Multiple R-squared and adjusted R-squared of the linear regression model based on natural logarithm of ETH-USD are larger than those of raw price. While in terms of F-statistics and p-value, all variables in both models are significant. And the abnormal numbers of raw price model also show that the model is far from being precise.
Thus, in this case, it's more accurate to model the natural logarithm of ETH-USD price as the outcome y.

Forward Selection

After the forward process, the selected 8 predictor variables are as below.

N	Explanatory variable
1	log (fiat volume lagged by one day)
2	Log (iShares MSCI USA Value Factor ETF)
3	Log (GBP/USD)
4	Log (IVW US Equity)
5	Log (T5YIE)
6	Log (Invesco DB Energy Fund)
7	Log (EUR/USD)
8	Log (TRC1(coal))

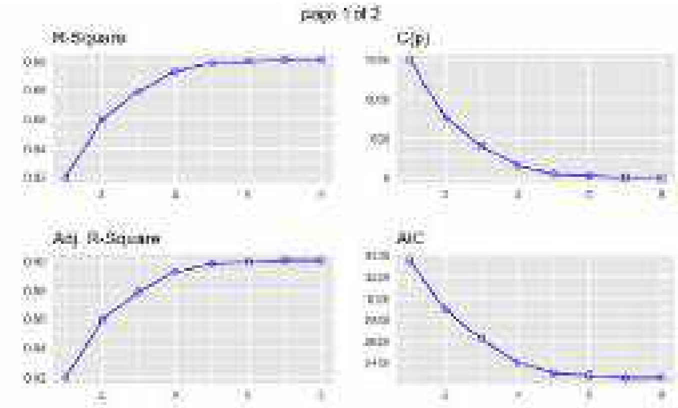


From R-squared and Adj R-squared charts, we can see that with increasing number of selected variables from 1 to 8, the eventually number is becoming larger, eventually reaching 0.90, which shows the best explanation power of ETH-USD is based on 8 selected variables.
From the Cp chart we can clearly see that with number of variables introduced into the model from 1 to 8, the Cp is getting closer to the number of predictors plus the constant, showing the model is getting more precise and unbiased.
In the last chart, a decreasing AIC score with more variables shows the linear regression model is performing better and better.

Best Subset

Here we analyse all the possible best performing subsets with the forward selected 8 regressors.
Below are example best subsets with 4 and 5 variables:

N	Subset with 4 variables	Subset with 5 variables
1	log (fiat volume lagged by one day)	log (fiat volume lagged by one day)
2	Log (IVW US Equity)	Log (IVW US Equity)
3	Log (Invesco DB Energy Fund)	Log (Invesco DB Energy Fund)
4	Log (EUR/USD)	Log (EUR/USD)
5		Log (T5YIE)



It is clear from the charts above that increasing the number of variables improves the quality of the model up to a point. From 6 variables to 8 variables, the R-squared only increases by less than 0.001, while the residual errors of forecast increase due to the additional variables introduced.
Thus, the selected the model is based on 5-variables best subset.

VIF

These are the 5 selected variables after sub-set process, and their VIF values are as below:

N	Explanatory variables	VIF Value
1	log (fiat volume lagged by one day)	3.57
2	Log (IVW US Equity)	2.24
3	Log (Invesco DB Energy Fund)	2.27
4	Log (EUR/USD)	2.56
5	Log (T5YIE)	2.78

No VIF values have exceeded the threshold of 5, meaning no variable has strong multicollinearity with 2 or more variables. So, no variable has to be eliminated from the final model.

Selected Model Summary

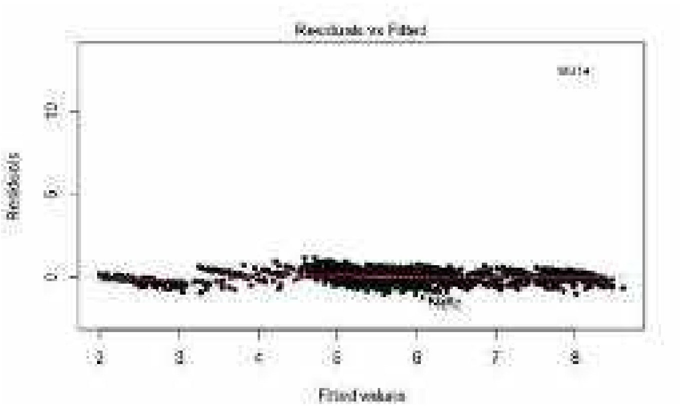
Coefficients:	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-12.64	0.290	-43.5	<2e-16
Log (fiat volume lagged by one day)	0.39	0.010	35.86	<2e-16
Log (EUR/USD)	9.05161	0.403	22.43	<2e-16
Log (IVW US Equity)	1.74	0.061	28.3	<2e-16
Log (T5YIE)	-0.61	0.059	-10.34	<2e-16
Log (Invesco DB Energy Fund)	1.51	0.082	18.28	<2e-16

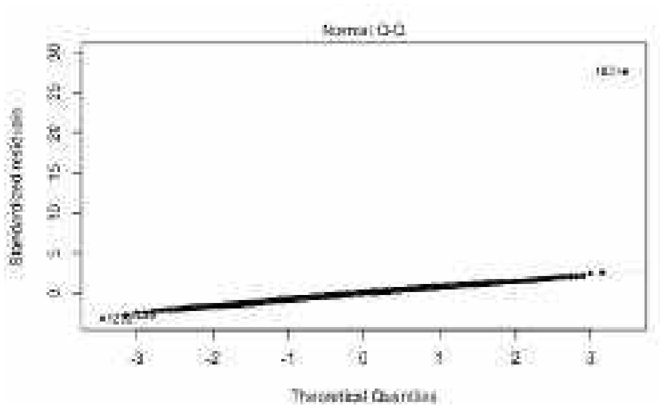
Model	Multiple R-squared	Adjusted R-squared	F-statistic	p-value
Final Log	0.8988	0.8986	3243	< 2.2e-16
Log	0.912	0.9106	680.3	< 2.2e-16

Linear Equation:

$$\log(ETH/USDt) = -12.64 + 0.39 * log(fiat\ volume\ lagged\ by\ one\ dayt) + 9.05 * log(\text{EUR/USD}t) + 1.74 * log(\text{IVW US Equity}t) + (-0.61) * log(\text{T5YIE}t) + 1.52 * log(\text{Invesco DB Energy Fund}t)$$

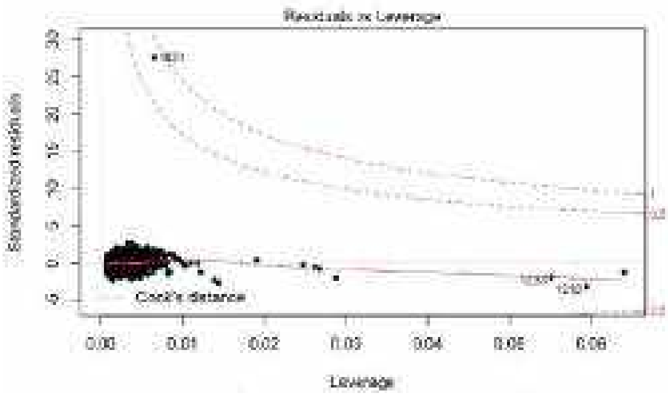
Overall based on the statistics numbers, both Multiple R2 and Adjusted R2 are close to 1, with a larger F-statistic and a smaller p-value, showing that the final model is statistically significant.

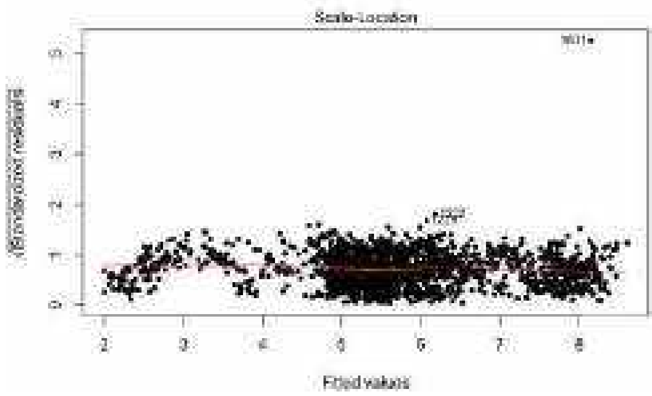




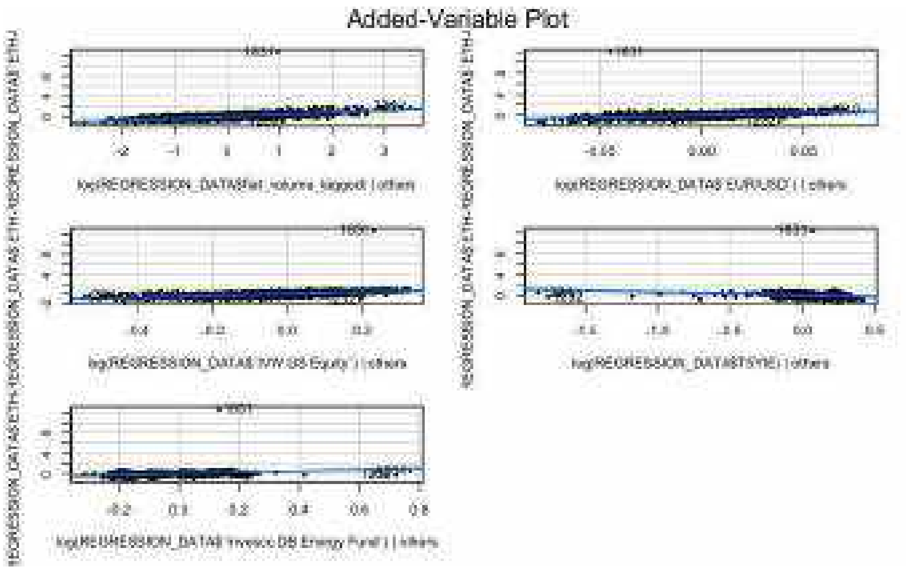
The residuals and fitted plot show a perfectly well-behaved result, because:

- The residuals "bounce perfectly" around the 0 line. This suggests that the assumption that the relationship is linear is reasonable.
- The residuals perfectly form a "horizontal band" around the 0 line. This suggests that the variances of the error terms are equal.
- One residual in 1831 observation "stands out" from the basic random pattern of residuals. This suggests that there are roughly no large outliers. As all the points fall perfectly along this reference line, we can assume normality.





As we here see a relative horizontal line with randomly spread points, it means that the model is relative accurate and of homoscedasticity. The last plot shows first, the spread of standardized residuals appears to be relative stable, confirming a linear relationship; second, there are 1 point that has high influence on the model if being deleted, because it's outside Cook's distance, which is the point with residual in 1831 observation.



These plots allow us to conveniently visualize the relationship between each individual predictor variable and the response variable. For example, in the charts we can find all but T5YIE are positively related with ETH-USD. And all their effects on ETH/USD are not too significant.

Model misspecification

Test for homoskedasticity and no correlation

White test

$$H_0: \sigma_i^2 = \sigma^2 \ H_a: \sigma_i^2 > \sigma^2$$

The result is W= 24.52 with a p-value of 0.006. Thus, we reject the null hypothesis highlighting the presence of heteroskedastic errors.

Breusch-Godfrey test

$$H_0: \sigma_i^2 = \sigma^2 \ H_a: \sigma_i^2 = \lambda + \delta Z_i$$

The result led to BPG= 202.67, p-value < 2.2e-16, thus we reject once again the null hypothesis.

Goldfeld-Quandt test

$$H_0: \sigma_i^2 = \sigma^2 \ H_a: \sigma_i^2 = cz_i^2$$

We test this statistic by ordering our samples on the basis of different explanatory variables.

Ranking variable	Fiat Volume lagged	EUR/USD	Invesco DB Energy	IVW US Equity	T5YIE
GQ statistic	3.31	0.56	2.39	1.97	3.36
p-value	2.2e-16	1	2.2e-16	2.2e-16	2.2e-16

Durbin Watson test

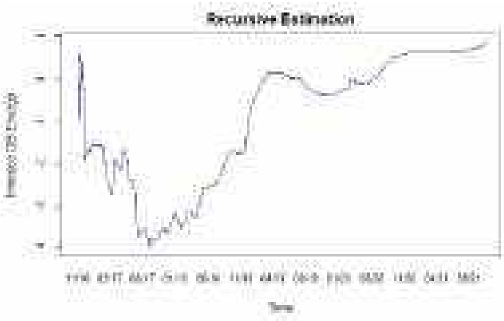
$$H_0: \varepsilon_i \ uncorrelated \ \ H_a: \varepsilon_i = \rho\varepsilon_{i-1} + u_i$$

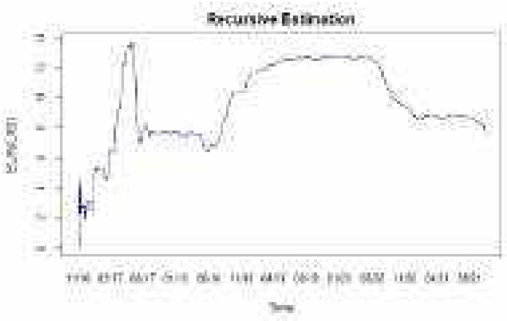
The result led to DW= 0.62, thus we reject the null hypothesis.

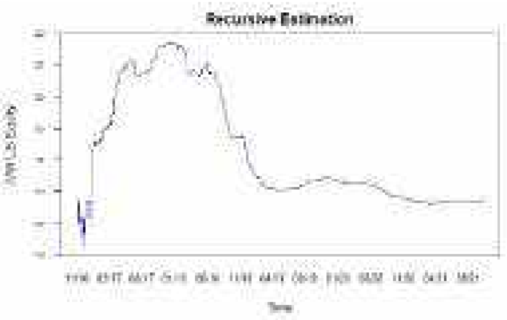
Test for parameter instability

Recursive estimation

The charts below show the parameters that are clearly not sable over time.

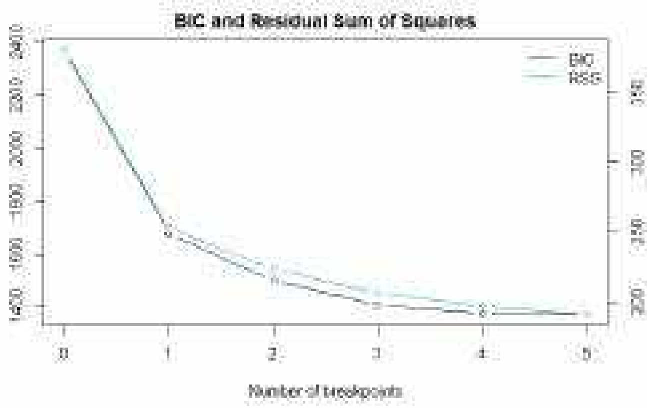






Bai-Perron test

$$H_0: no\ breaks \quad H_a: m\ breaks$$



As it is clear from the chart above the performance of the model significantly improves if we consider 4 structural breaks. These are 18/02/2018, 19/11/2018, 21/11/2019, 04/11/2020.

Chow test

$$H_0: \beta_1 = \beta_2 \; H_a: \; \beta_1 \neq \beta_2$$

Break date	18/02/2018	19/11/2018	21/11/2019	04/11/2020
CH statistic	65.01	111.36	82.68	60.33
p-value	2.2e-16	2.2e-16	2.2e-16	2.2e-16

Dummy Variables

In the table below are stated the main statistic of the new model with the inclusion of dummy variables to counter parameter instability.

Coefficients	Estimate	Std. Error	t value	PR(>|t|)
(Intercept)	-17.235	0.528	-32.64	< 2e-16
Fiat Volume lagged	0.3973	0.011	35.66	< 2e-16
EUR/USD	6.7544	0.437	15.43	< 2e-16
Invesco DB Energy	1.322	0.128	10.27	< 2e-16
IVW US Equity	3.455	0.168	20.45	< 2e-16
T5YIE	-0.607	0.070	-8.59	< 2e-16
dummy1	-0.101	0.044	-2.28	0.02

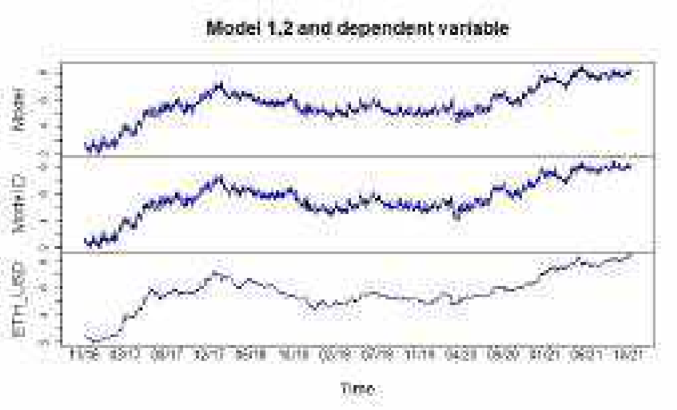
dummy2	-0.483	0.047	-10.11	< 2e-16
dummy3	-0.248	0.055	-4.43	9.60E-06
dummy4	-0.149	0.064	-2.31	0.02

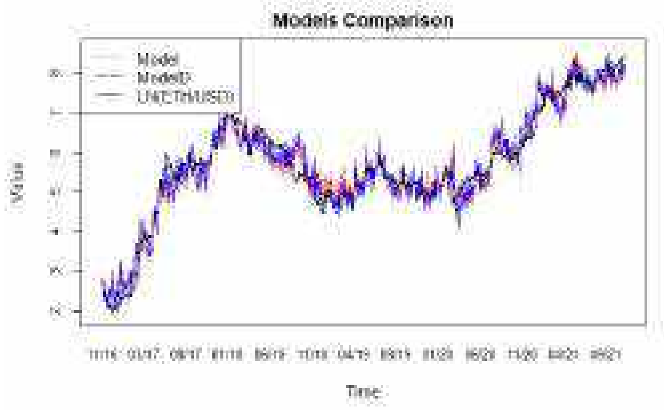
Multiple R-squared	Adjusted R-squared	F-statistic	p-value
0.9429	0.9426	3341	2.2e-16

Models' comparison

Graphical Output

To complete the analysis two graphical output are displayed to assess on a visual basis the performance of the model both with and without dummies in respect to our independent variable (ln(ETH/USD)).





Diebold Mariano test

$$H_0: model\ is\ accurate\ as\ model\ with\ dummies\ H_a:\ \ model\ is\ less\ accurate\ than\ model\ with\ dummies$$

Loss Function	DM statistic	p-value
MSE	4.4749	3.823e-06
MAE	5.1409	1.367e-07

Conclusion

The introduction of dummies clearly enhances the model ability to predict the independent variable. Although the model has some significant limitations it allows us to draw some conclusions:

1. In this model the addition of dummies clearly enhances the forecast quality. This is both true at MAE and MSE level. However, the large spike in returns between October and November 2021 is not captured by the model. Indeed, the overall adjusted R^2 reaches 0.9046.
2. The returns of ETH/USD are significantly correlated with EUR/USD, with a coefficient of 6.75444. This comes as no surprise since EUR/USD is regarded as a risk on trade.
3. Ethereum returns are significantly correlated with IVW US Equity, an index capturing US growth stocks. This confirms that also Ethereum performs better when risk aversion is lower.
4. Ethereum is negatively correlated with T5YIE, thus also Ethereum fails to appear as an inflation hedge.
5. Changes in fiat Volume lagged by one day are positively correlated with Ethereum returns. This shows how this Ethereum price is also liquidty driven.

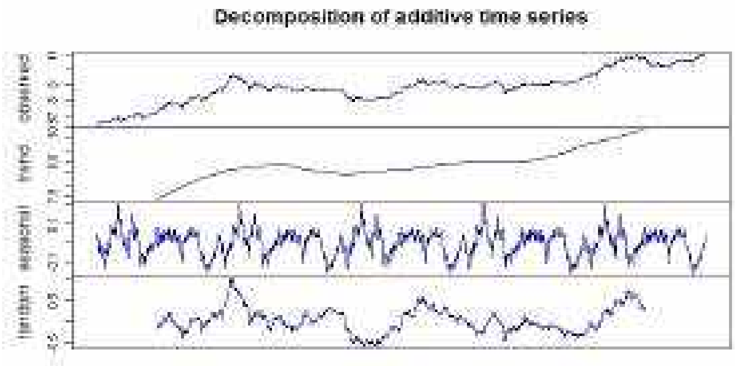
FITTING ARMA MODEL

Dynamic linear models may prove a more flexible tool to model Ethereum and Bitcoin. Indeed, the basic idea of univariate dynamic linear model is to use the past behaviour of the variable as an explanatory variable. We will focus our attention on ARMA models a particular family of dynamic linear models, these models require the time series to be weakly stationary. This means that the moments of variable remain constant throughout time.

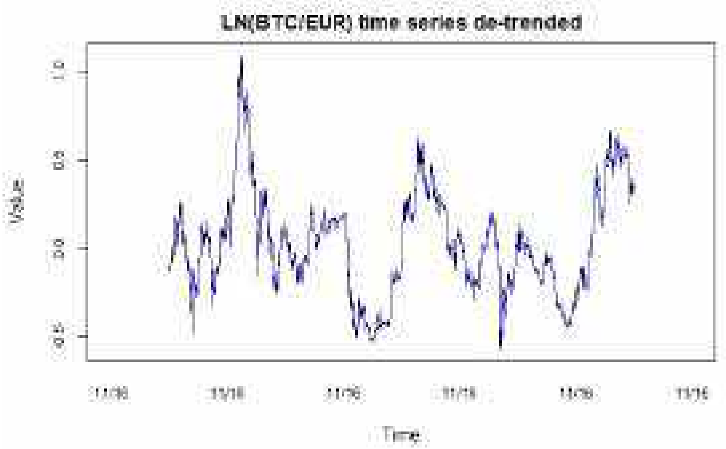
Stationarity

Plotting the BTC/EUR time series it can be clearly assessed the utter absence of stationarity.

Thus, a first approach rest in decomposing the time series through additive components. This procedure is implemented on logarithm rather than prices. The output below shows the different component of the univariate time series.



A first approach to produce a stationary time series is done by detrending the logarithm of BTC/EUR.

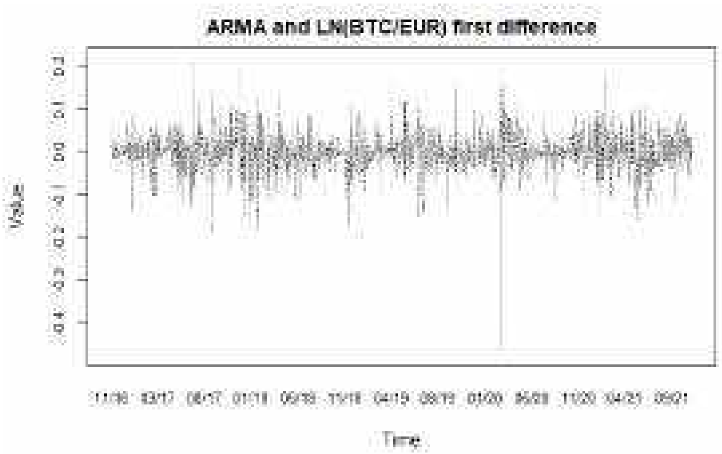


Then two widely accepted test on stationarity are performed. The Ljung Box Q test, that follows a chi square distribution and the augmented Dickey and Fuller test asymptotically distributed according to t statistic. In the former test the null does not reject stationarity (accept), while in the latter the alternative hypothesis is stationarity. The result is summarized in the table below, it is clear that the time series manipulation does not lead to stationarity.

Test	Statistic	P-value
Ljung Box Q test	4221.7	2.2e-16
Augmented Dickey Fuller test	-2.93	0.1811

AC-PAC based specification

Failing to produce stationarity by time series manipulation a more widely accepted and mechanical process is employed. The correlograms of the returns of BTC/EUR are plotted. This allows us to identify the presence of a unit root since PAC (1) ≈1. Consequently, an ARIMA model of order one is specified leading to the below result.



Although apparently the model correctly predicts the time series the specification is quite useless. Indeed, the model is an ARIMA (0,1,1), is a white noise over the first difference of BTC/EUR return. Failing to have any significant partial autocorrelation coefficient does not allow us to speculate on a particular serial correlation structure of the time series. Similar conclusions apply to the pairs BTC/USD, ETH/EUR, and ETH/USD.

Disclaimer

This research material has been prepared by Minerva Invest. Minerva Invest specifically prohibits the redistribution of this material in whole or in part without the written permission of Minerva Invest. The research officer(s) primarily responsible for the content of this research material, in whole or in part, certifies that their views are accurately expressed, and they will not receive direct or indirect compensation in exchange for expressing specific recommendations or views in this research material. Whilst we have taken all reasonable care to ensure that the information contained in this publication is not untrue or misleading at the time of publication, we cannot guarantee its accuracy or completeness, and you should not act on it without first independently verifying its contents. Any opinion or estimate contained in this report is subject to change without notice. We have not given any consideration to, and we have not made any investigation of the investment objectives, financial situation or particular needs of the recipient or any class of persons, and accordingly, no warranty whatsoever is given and no liability whatsoever is accepted for any loss arising whether directly or indirectly as a result of the recipient or any class of persons acting on such information or opinion or estimate. You may wish to seek advice from a financial adviser regarding the suitability of the securities mentioned herein, taking into consideration your investment objectives, financial situation, or particular needs, before making a commitment to invest in the securities. This report is published solely for information purposes, it does not constitute an advertisement and is not to be construed as a solicitation or an offer to buy or sell any securities or related financial instruments. No representation or warranty, either expressed or implied, is provided in relation to the accuracy, completeness or reliability of the information contained herein. The research material should not be regarded by recipients as a substitute for the exercise of their own judgement. Any opinions expressed in this research material are subject to change without notice.